KRISHNASAMY COLLEGE OF SCIENCE, ARTS & MANAGEMENT FOR WOMEN

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DEPARTMENT OF MATHEMATICS

QUESTION BANK 2021-2022

I- M.SC MATHEMATICS

SEMESTER I

ALGEBRA

UNIT – I

6 Marks Questions

1. P.T Conjucacey is an equalence relation.

2. Define Normalizer and P.T the normalicer N (a) is a sub group of G.

3. If G is a finite, then C_a = O(G) / O(N(a)).

4. P.T O(G) = $\sum {O(G)}/{O(N(a))}$.

5. Define center of group and PT the center if group is normal subgroup of G.

6. a$\in Z$ iff N(a) = G.

7. f O(G) = p^n where p is a prime number then Z(G)$\neq \{e\}$

8. If O(G) = P^2 where P is prime number then G is abelian.

9. If P is a prime number and P/O(G) then G has an element of order P.

10. The number conjucate in S_n is P(n) where P(n) is the number of partition of n.

11. If H is P – sylow sub group of G and x$\in G$ the x^{-1} H x is also P- sylow subgroup of G.

12. The Equalence relation on AxB={axb / a$\in A, b \in B$}.

13. Let A and B are finite sub group of G then $O(G) = {O(A)O(B)}/{O(A \cap xBx^{-1})}$.

14. If H and K are finite subgroup of G and O(H) and O(K) respectively then

$O(HK) = {O(H)O(K)}/{O(H \cap K)}$.

15. The number P- sylow subgroup in Gfor a given prime number P is of the form 1+Kp.

16. Let G be a group of order $11^2 . 13^2$ .then find how many 11 and 13 sylow subgroup there are in G.

17. If G is a abelian group then C_a= 1.

18. if G is finite. a $\in Z$ iff O(N(a)) = O(G).

19. The permutation (1,2) in $S_n$what elements commutes with it ?.

20. If $p^m/_{O(G)}$and $p^{m+1} \nmid$ O(G). then G has a sub grouop of order p^m.

15 MARKS QUESTIONS

1. Derive class equation.

2. State and Prove I st part of Sylows Theorem.

3. If P is a prime number and P/O(G) then G has an element of order P.

4. IF O(G) = p^n where p is a prime number then Z(G)$\neq \{e\}$

5. If O(G) = P^2 where P is prime number then G is abelian.

6. . If G is a finite, then C_a = O(G) / O(N(a)). And also P.T O(G) = $\sum O(G)/_{O(N(a))}$.

7. . The number P- sylow subgroup in G for a given prime number P is of the form 1+Kp.

Let G be a group of order $11^2 . 13^2$ .then find how many 11 and 13 sylow subgroup there are in G.

8. State and Prove third part of Sylows theorem.

9. The number P- sylow subgroup in G for a given prime number P is of the form 1+Kp.

then find, Let G be a group of order $11^2 . 13^2$ .then find how many 11 and 13 sylow subgroup there are in G.

UNIT II

6 MARKS QUESTIONS

1. Define solvable group and P.T Every Abeliann group is solvable.

2. If G is Abelian then G' = {e].

3. G' is normal subgroup of G.

4. G/ G' is abelian.

5. If H is subgroup of G and H contains $G^{/}$.

6. Let M be a n0ormal subgroup of G such that G/M is abelian then M contains $G^{/}$.

7. If G is solvable then $G^{(k)} = \{e\}$. for some int k.

8. Every Homomorphic image of a solvable group is solvable.

9. Let G= S_n where n$\geq$ 5. then G^k for k = 1,2,3,… contains every 3- cycle of S_n.

10. If A and B are any 2 groups then $A \times B$ is a group under the operation defined by

$(a_1, b_1)(a_2, b_2) = (a_1 a_2, b_1 b_2)$.

11.Supose that G is Internal direct product of N_1 ,N_2 , …… N_n.

then for each i$\neq j, N_i \cap N_j = \{e\}$

12. Let G be a group and suppose that G nis the internal direct product of N_1 ,N_2 , …… N_n.. let

T = $N_1 \times N_2 \times \ldots\ldots \times N_n$. then G and T are are isomorphic.

13. Define Module . If T is a homomorphism of M into N let K(T) = {x$\in / \ xt = 0$}. P.T K(T)is a sub module of M and that I(T) = {xt/ $x \in M$} is sub module of N.

14. P.T T is a isomorphism iff K(T)=0.

15. If A and B is sub module of M P.T

I. A $\cap B$ is a sub module of M.
II. A+B = { a+b $/a \in A, b \in B$}.

16. If M is an module of and A is a sub module of M, define Quotient module M/A. S.T if it's a Sub module of M and

17. If $G^{/}$ = {e] then G is Abelian.

18. If $G^{(k)} = \{e\}$. then G is solvable for some int k.

19. Supose that G is Internal direct product of N1 , N2 , … … Nn. $if a \in N_i$ and $b \in N_j$.then ab = ba.

20. If M is an module of and A is a sub module of M, P.T there is an R – homomorphism of M into M/A.

15 MARKS QUESTIONS

1. S_n is not bsolvable for $n \geq 5$.

2. Every finite theorem of finite abeliann group is the direct product of cyclic group.

3. Let R be a Euclidean ring then any finitely generated R module M is the direct sum of a finite number of cyclic sub modules.

4. Any finite abelian group is the direct product of a cyclic groups.

5. G is solvable iff $G^{(k)} = \{e\}$. for some int k.

6. Let G= S_n where n≥ 5. then G^k for k = 1,2,3,… contains every 3- cycle of S_n.

7. Define solvable group, and derive the the result about G.

8. (a)Supose that G is Internal direct product of N_1 ,N_2 , …… N_n.

then for each i$\neq j, N_i \cap N_j = \{e\}$ and Supose that G is Internal direct product of N1 , N2 , … … Nn.$if a \in N_i$ and $b \in N_j$.then ab = ba.

(b)Let G be a group and suppose that G n is the internal direct product of N_1 ,N_2 , …… N_n.. let

T = $N_1 \times N_2 \times \ldots\ldots \times N_n$. then G and T are are isomorphic.

UNIT III

6 MARKS QUESTIONS

1. If W$\subset V$ is invariant under T, then T induces a linear tansformation $\bar{T} on \,{}^{V}/_{W}$ defined by

(v+W)$\bar{T}$= vT + W. If T satisfies the minimal polynomial q(x) ∈ f(x) so does $\bar{T}$ if P_1 (x) is the minimal

polynomial $\bar{T}$ over F and if P(x) is that of T , then P_1 (x) / P(x).

2. If V is n- dimensional over F and if T $\in A(V)$ has all its characteristic roots in F_1 then satisfies a polynomial of deg n over F.

3. If V = $V_1 \oplus V_2 \oplus \ldots\ldots . \oplus V_k$, where each subspace v_i is of dimension v_i and is invariant under T∈ A(V) then a basis of V can be found so that the matrix of T in this basis of the form $\begin{pmatrix} A_1 \ldots\ldots. & 0 & 0 \\ 0 & A_2 \ldots .. & 0 \\ 0 & 0 \ldots\ldots. & A_k \end{pmatrix}$.

4. If T $\in A(V)$ is nilpotent then $\propto_{\circ} + \propto_1 T + \ldots. + \propto_m T^m$. is invertible if $\propto_{\circ} \neq 0$.

5.If u ∈v_1 such that $uT^{n_1} - k = 0$, where $0 \le k \le n$, then u = $u_o T^k$ where $u_o \in v_1$.

6. If V = $V_1 \oplus V_2$, where If V = $V_1 and V_2$ are subsapace of dim n_1 and dim n_2 and are invarient under T.

7. If M of dimension m_1 is cyclic w.r.t T then the dimension of MT^k is m-k for all k$< m$.

8. The invariants T are unique .

9. If two nilpotent transformation are similar then they have the same invariants .

10. If they have same invariants then they are similar.

11. If S and T are nilpotent transformation provbe that S.T and S+ T are nilpotent .

13. Prove that $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent find its invariants.

14. Suppose A is nilpotent of index k. show that A^n , n is nilpotent of index K.

15. Prove that the relation of similarity is an equalence relation.

15 MARKS QUESTIONS

1. If T ∈ $A(V)$ is nilpotent of index of V can be found such that matrix of T isw the basis has the form $\begin{pmatrix} M_{n1} \dots\dots & 0 & 0 \\ 0 & M_{n2} \dots\dots & 0 \\ 0 & 0 \dots\dots & M_{nr} \end{pmatrix}$. where $n_1 \geq n_2 \geq \cdots \geq n_r$.

2. If T ∈ A(V) has all its cgharacteristic roots in F show that there exists a basis of V in which the matrix of T is triangular.

3. If W⊂ V is invariant under T, then T induces a linear tansformation $\bar{T} on \, ^V/_W$ defined by

(v+W)$\bar{T}$= vT + W. If T satisfies the minimal polynomial q(x) ∈ f(x) so does $\bar{T}$ if P_1 (x) is the minimal

polynomial $\bar{T}$ over F and if P(x) is that of T , then P_1 (x) / P(x).

4. Two nilpotent transformation are similar iff they have the same invariants .

5.If V is n- dimensional over F and if T ∈ $A(V)$ has all its characteristic roots in F_1 then satisfies a polynomial of deg n over F.

6. If V = $V_1 \oplus V_2 \oplus \dots \dots . \oplus V_k$, where each subspace v_i is of dimension v_i and is invariant under T∈ A(V) then a basis of V can be found so that the matrix of T in this basis of the form $\begin{pmatrix} A_1 \dots \dots . & 0 & 0 \\ 0 & A_2 \dots . . & 0 \\ 0 & 0 \dots \dots . & A_k \end{pmatrix}$.

where each A_i is an $n_i \times n_i$ matrix and is the matrix of the linear transformation induced by T on V_i.

7. Define nilpotent transformation and invariants and prove that Prove that $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent find its invariants.

8. (i) If V = $V_1 \oplus V_2$, where If V = $V_1 and V_2$ are subsapace of dim n_1 and dim n_2 and are invarient under T.

(ii) If M of dimension m_1 is cyclic w.r.t T then the dimension of MT^k is m-k for all k$< m$.

UNIT IV

6 MARKS QUESTIONS

1. Define a decomposition of V. and Let g(x) ∈ F(x) the linear transformation induced by d(t) on V_1. is $g(T_1)$.

2. If the minimal polynomial of T_1 and T_2 over F $P_1(x)$ and that of $P_2(x)$ then minimal polynomial pf T over F is the LCM of $P_1(x)$ and $P_2(x)$.

3. If of $P_1(x)$ where each V_i is invariant under T and if $P_i(x)$ is the min polynomial of T_i, the linear transformation induced by T over V_i then min polynomial of T is the least common multiple of of $P_1(x) .P_2(x) \dots . P_k(x)$

For each I = 1,2,3,…k, $V_i \neq 0$ and V = $V_1 \oplus V_2 \oplus \dots \dots . \oplus V_k$,, the min polynomial of T_i is $q_i (x)^{li}$.

5. Let T ∈$A_F(V)$ have all its distinct characteristic roots $\lambda_1 \lambda_1, \lambda_2, \dots . \lambda_k$ in F then the basis of V can be found in which the of T is of the form $\left(\begin{pmatrix} J_1 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & J_1 \end{pmatrix} \right)$ wher each J_i = $\begin{pmatrix} B_{i1} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & B_{iri} \end{pmatrix}$ and where B_{i1}, B_{i2} … B_{iri}. are basic Jordon blocks belonging to λ_i.

6. Write down the basis Jordon blockn belonging to the characteristic root λ= 5, λ= -1, λ=0, of order

1,2,3,4…

7. Prove that the matrix A =$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ is nilpotent.

8. Prove that the matrix A =$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ is invariant.

9. Prove that the matrix A =$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ is Jordon form.

10. Prove that the matrix A =$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ is not similar.

11. Define Similarity transformation and S.T similae is an equation.

12. If P(x) is the minimal polynomial of T ∈ A(V) and if V as a module of cyclic module method of related to T S.T there exists a basic of V over F in which the matrix of T is C[P(x)] the campanion matrix of P(x).

UNIT V

6 MARKS

1. P.T If T∈ A(V) ,then given any vV,there exists an element .w∈ V depending on v and T (uT,v)=(u,w) for every u∈ V. this element w is uniquely determined by v and T.

2. If T ∈A(v) then T^* ∈A(v) .Moreover (i)$(T^*)^*$ = T (ii) $(S+T)^* = S^* + T^*$ (iii) $(\lambda S)^*=\bar{\lambda}S^*$ (iv) $(ST)^* = T^*S^*$ for every S,T ∈ A(v).
3. PT if T∈ A(V) is unitary if and only if TT^*=1.

4. If {v_1,v_2,…..v_n} is an orthogonal basis of V and if the matrix of T∈ A(V) in this basis is (α_{ij}) then the matrix of in this basis is(β_{ij}) where β_{ij}=$\overline{\alpha_{ij}}$.

5.PT If T∈ A(V) is Hermition then all its characteristic roots are real.
6. PT if S∈ A(V) and if VSS^* then vs=0.
7. PT if N is a normal linear transformation and if vN=0,for v∈ V then vN^*=0.
8. PT if T is unitary and if λ is a characteristic root of T then| λ|=1.

9. PT If N is a normal and if $vN^k=0$ then vN=0.
10. PT if N is anormal and if for $\lambda \in F$,$v(N-\lambda)^k=0$ then $vN=\lambda$ v.

15MARKS

1. PT if N is a normal L T on V then there exists an orthonormal basis consisting of characteristic vectors of N ,in which the matrix of N is diagonal .

2.PT the normal transformation N is (i) Hermition if and only if it's characteristic roots are real.(ii) Unitary if and only if it's characteristic roots are aii of absolute value.

3. PT If N is normal and AN=NA then $AN^*=N^*A$.

4. PT the hermition linear transformation T is nonnegative if and only if all of its characteristic roots are nonnegative.

5. PT $T\geq 0$ ifand only if $T=AA^*$ for some A.

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REAL ANALYSIS I

SECTION – A

UNIT-I

1. Define Monotonic function.
2. Define jump of f.
3. Define partition.
4. Define bounded variation.
5. Define total variation.
6. State additive property of total variation.

UNIT-II

1. Define Riemann stieljes integral.
2. Define refinement of a partition.
3. Define norm partition.
4. Define Riemann stieljes sum.
5. Define Riemann integrable.
6. Definestep function.
7. State Euler summation formula.
8. Define greatest integer function.
9. Define upper & lower stieljes sum.
10. Define upper & lower stieljes integral.

11. State Riemann condition.

UNIT-III

1. State necessary condition for existence of Riemann stieljes integrals.
2. State sufficient condition for existence of Riemann stieljes integrals.
3. State Mean value theorem for Riemann stieljes integral.
4. State second Mean value theorem for Riemann stieljes integral.
5. State second fundamental theorem for integral calculus.
6. State change of variable in Riemann integral.
7. State second Mean value theorem for Riemann integral.

UNIT-IV

1. Define convergence.
2. Define limit of a function.
3. Define Cauchy sequence.
4. Define infinite series.
5. Define telescopic series.
6. Define Cauchy condition.
7. Define absolute convergent & conditional convergent.
8. State Dirichlet's test.
9. State Abel's test.
10. Define double sequence.
11. Define double series.
12. Define multiplication of series.
13. State Merten's theorem.
14. Define Cesarosummable.
15. Define infinite product.
16. State Cauchy condition for products.

UNIT - V

1. Define pointwise convergence of sequences of functions.
2. Define uniform convergence.
3. Define uniform boundedness.
4. State Cauchy condition for uniform convergence of sequence.
5. Define uniform convergence of infinite series of a function.
6. State Cauchy condition for uniform convergence of series.
7. State Weierstrass M test.
8. State Dirichlet's test for uniform convergence.
9. State Mean convergence.

SECTION – B

UNIT-I

1. Define Monotonic function .Explain its properties?
2. If f is monotonic on a closed interval [a,b]then show that the set of discontinuities of f is countable.
3. If f is monotonic on [a, b] then show that f is of bounded variation.
4. If f is continuous on [a,b] and if f' exist and it is bounded in the interior then show that f is of bounded variation on [a, b].
5. If f is a bounded variation on [a, b] i.e, $\sum_{k=1}^{n} |\Delta f_k| \leq M$ for every partition P[a,b] then prove that f is bounded on [a, b] and also |f(x)|≤ |f(a)|+M , ∀x∈ [a, b].
6. If f and g are each of bounded variation on [a, b] , then prove that their sum, difference, and product are also of bounded variation on [a ,b] also prove that $V_{f+g} \leq V_f$+V_g, and $V_{f.g} \leq A\, V_f + BV_g$, where A= sup {|g(x)|:x∈[a, b]}and B=sup {|f(x)|:x∈[a, b] }.
7. Let f be defined on [a, b].prove that f is a bounded variation on [a, b] if and only if, f can be expressed as the difference of two increasing functions.

UNIT- II

1. Define (i) Refinement of p ,(ii)Norm partition,(iii) R.S. integral, (iv) Riemann integrable.
2. If f ∈R(α)$, and\ g \in R(\beta)$ on [a, b] then($C_1 f + C_2 g$) ∈ R(α) on [a, b], where $C_1\ and\ C_2$ are constants. Also$\int_a^b (C_1 f + C_2 g\ )$ dα =$C_1 \int_a^b f\ d\alpha + C_2 \int_a^b g d\alpha$.
3. If f ∈R(α)$, and\ f \in R(\beta)$ on [a, b] thenf∈ $R(C_1\alpha + C_2\beta)$ ∈ R(α) , where $C_1\ and\ C_2$ are constants. Also $fd(C_1\alpha + C_2\beta\ )$ dα =$C_1 \int_a^b f\ d\alpha + C_2 \int_a^b f d\beta$
4. 4. Assume that c ∈ $[a, b]\ if\ two\ of\ the\ three\ integrals\ in\ the\ equation\ \int_a^c f\ d\alpha + \int_c^b f\ d\alpha = \int_a^b f\ d\alpha$.
5. Let f be defined on [a, b] in such a way that at least one function f or αwhich is continuous from the left at c and atleast one is continuous from right at c, then prove that f∈ $R(\alpha) on$ [a, b] also we have $\int_a^b f\ d\alpha = f(c)[\alpha(c^+) - \alpha(c^-)]$.
6. State and prove Euler summation formula.
7. Assume that α is ↑ on [a, b] then prove that (i) If $P' \underline{C}$ P, then we have U(P',f,α)≥L(P,f, α) and L(P',f, α))≥L(P, f, α) and (ii)For any two partitions P_1 and P_2 , we have L(P_1,f, α)≤U(P_2 , f, α).
8. Assume that $\alpha$is increasing on [a, b] then prove that $\underline{I}$(f, α) ≤ $\bar{I}$(f, α).
9. State and prove additive and linearity properties of upper and lower steigiles integral.
10. Give an example to show that $\underline{I}$(f, α) ≤ $\bar{I}$(f, α).
11. 11. If f∈ $R(\alpha) on$ [a, b] and α is increasing on [a, b] then prove that| f|∈ $R(\alpha) on$ [a, b].

UNIT- III

1. State and prove second fundamental theorem of integral calculus.
2. If is continuous on [a, b] and if a is ;of bounded variation [a, b] then prove that f∈R(α) on [a, b].
3. State and prove the first mean-value theorem for Riemann- Stieltjes integrals.
4. State and prove a necessary condition for existence of Riemann- Stieltjes integrals.
5. Let α be of bounded variation on [a, b] and if $f \in R(\alpha) on$ [a, b], Prove that $f \in R(\alpha)$on every subinterval [c, d] of [a, b].
6. State and prove Bonnet's theorem.
7. Establish the theorem on change of variable in a Riemann integral.
8. Assume that $f \in R(\alpha) on$ [a, b], let α be a function which is continuous on [a, b] and whose derivative $\alpha'$is Riemann integrable on [a, b] then prove that the following integral exists and are equal.
9. Let F is continuous at each point (x, y) of rectangle Q= {(x, y), a$\leq x \leq b$,c$\leq y \leq d\}$. Assume that α be of bounded variation on [a, b] and Let F be a function defined on [c, d] by the equation F(y)= $\int_a^b f(x, y) d\alpha(x)$then prove that f is continuous on [c, d] .
10. If f is continuous on the rectangle [a, b] X [c, d] if g∈R on [a,b] then for F is defined by the equation i.e, if $y_o \in$ [c, d].We have $\lim_{y \to y_0} \int_a^b g(x) f(x, y) dx = \int_a^b g(x) f(x, y_o) dx$

UNIT-IV

1. State and prove Merten's theorem.
2. Prove that the series $\sum a_n b_n$ converges if $\sum a_n$ converges and if $\{b_n\}$ is a monotonic
3. Prove the absolute convergence of $\sum a_n$ implies convergence.
4. Let $\sum a_n$ be a series of complex terms whose partial sums form a bounded sequence. Let
5. $\{b_n\}$be a decreasing sequence which converges to 0. Prove that $\sum a_n b_n$ converges.
6. If a series is convergent with sum s, show that it is also (C,1) summable with cesaro
   a. sum s.
7. If f(x) = $\sum_{n=0}^{\infty} a_n x^n$, for -1$< x < 1$and $\lim_{n \to \infty} na^n = 0$ and if f(x)→s as x→ $1 -$, show
   a. that$\sum_{n=0}^{\infty} a_n$, converges and has sum s.
8. Prove that the converse of absolute convergence of $\sum a_n$ implies convergencewith an
   a. example.
9. State and prove Dirichlet's test.
10. State and prove Able's test.
11. Let $\sum a_n$ be an absolutely convergent serieshaving sum s then Re-arrangement of
    a. $\sum a_n$ also converges absolutely and has the sum s.
12. State and prove the sufficient condition for equality of iterated series.
13. State and prove Cauchy condition for products.
14. Assume that each a_n >0 then prove that $\prod(1 + a_{n)}$ is converges if and only if the series

a. Let $\sum a_n$ converges.

15. Prove that Absolute convergence of $\prod(1 + a_{n)}$ implies convergence of $\prod(1 + a_{n)}$.
16. Assume that each $a_n \geq 0$then P.T $\prod(1 - a_{n)}$ converges if and only if the series $\sum a_n$ converges.

UNIT-V

1. State and prove Cauchy condition for uniform convergence of series.
2. If $\lim_{n\to\infty} f_{n=f,} \lim_{n\to\infty} g_{n=g,}$ on [a, b] and h(x)= $\int_0^x f(t)g(t)dt, h_{n(x)=} \int_a^x f_n(t)g_n(t)dt, for\ all\ x \in [a,b]$, then prove that $h_n \to h$ uniformly on [a, b].
3. Assume that f_n →f uniformly on S. If each f_n is continuous at a point c of S, then prove that the limit function f is also continuous at c.
4. State and prove Weierstrass M-test.
5. Let αbe of bounded variation on [a, b]. Assume that each term of the sequence $\{f_n\}$ is a real valued function such that f_n ∈R(α) on [a, b] for each n=1,2,3,….. Assume that f_n →f uniformly on [a, b] and define g_n(x)=$\int_a^x f_n$(t) dα(t), if x$\epsilon[a, b]$, n=1, 2,3,…. Prove that (i) f∈R(α) on [a, b], (ii) g_n →g uniformly on [a, b] where g(x)=$\int_a^x f(t)d\alpha(t)$.
6. State and prove Cauchy condition for uniform convergence of sequence.
7. Show that uniformly convergent sequence can be integrated term by term.
8. State and proveDirichlet's test for uniform convergence of series.

SECTION C

UNIT - I

1. State and prove additive property of total variation.
2. Let 'f' be of bounded variation on (a, b). If x ∈ (a, b) let V(x)=V_f(a, x) and V(a)=0. Then prove that every point of continuity of f is also a point of continuity of V. Also prove the converse is also true.
3. Let f be of bounded variation on [a, b] and c∈(a, b). Prove that f is of bounded variation on [a, c] and on [c, b] and $V_f(a,b) = V_f$(a, c) +$V_f(c,b)$.
4. 4. Let f be of bounded variation on [a, b]. Let V be defined on [a, b] as follows V(x) =$V_f(a,x), if\ a < x \leq b$,V(a)= 0 then prove that , (i)V is an increasing function on [a, b], (ii) V-f is an increasing function on [a, b].

UNIT- II

1. State and prove the theorem on change of variable in a Riemann- Stieltjes integral.
2. If f $\in R(\alpha) on$ [a, b], then prove that $\alpha \in$ R(f) on [a, b] and $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$.

3. State and prove Integration by parts formula for Riemann-Stieltjes integrals.
4. State and prove Euler summation formula.
5. If f$\in R(\alpha) on$ [a, b],and α has a continuous derivative
6. $\alpha' on\ [a,b], show\ that\ the\ Riemann\ integral\ \int_a^b f(x)\,\alpha'(x)dx$ existsand $\int_a^b f(x)d\,\alpha(x)$ $=\int_a^b f(x)\,\alpha'(x)dx$.
7. If α is increasing on [a, b] then prove that the following three statements are equivalent, (i) $f \in R(\alpha) on$ [a, b], (ii) f satisfies the Riemann condition with respect to α on [a, b]. (iii) $\underline{I}$(f, α) $\leq \bar{I}$(f, α).

UNIT - III

1. Assume that $\alpha$is of bounded variation on [a, b] .Let v(x) denote total variation of αon [a, x] if a$< x \leq b\ and$ let V(a)=0, Let f be defined and bounded on [a, b]. If f ∈R(α) on [a, b], then prove that f∈R(V) on [a, b].
2. State and prove second mean value theorem for Riemann integrals.
3. State and prove second fundamental theorem of integral calculus.
4. State and prove the necessary and sufficient condition for the existence of R.S. integral.
5. State and prove the first and second Mean value theorem for R.S integrals.
6. Assume f ∈R(α) on [a, b], and g ∈R(α) on [a, b].define F(x)=$\int_a^x f(t)d\alpha(t) and\ G(x) = \int_a^x g(t)d\alpha(t)$. If $\alpha \in [a,b]$then prove that f$\in R(g) and\ g \in R(F) and\ fg \in R(\alpha)$ on [a, b].

UNIT -IV

1. Ass;ume that $\sum_{n=0}^{\infty} a_n$,converges absolutely and has sum A, $\sum_{n=0}^{\infty} b_n$, converges with sum B. Then prove that the Cauchy product of these two series converges and has the sum AB.
2. State and prove Merten's theorem.
3. State and prove Cauchy condition for products.
4. If each a_n >0, and the product $\prod(1 + a_{n)}$converges , show that the series $\sum a_n$ converges and conversely.
5. State and prove Dirichlet's test and Able's test.
6. State and prove Rearrangement theorem for Double series.
7. State and prove the sufficient condition for equality of iterated series.

UNIT – V

1. Let α be of bounded variation on [a, b]. Assume that each term of the sequence $\{f_n\}$ is a real valued function such that $f_n \in R(\alpha)$ on [a, b] for each n=1,2,3,….. Assume that $f_n \to f$ uniformly on [a, b] and define $g_n(x)=\int_a^x f_n(t)\, d\alpha(t)$, if $x \epsilon [a, b]$, n=1, 2,3,…. Prove that (i) $f \in R(\alpha)$ on [a, b], (ii) $g_n \to g$ uniformly on [a, b] where $g(x)=\int_a^x f(t) d\alpha(t)$.
2. State and prove Dirichlet's test for uniform convergence of series
3. State and prove Weierstrass M-test.
4. State and prove Cauchy condition for uniform convergence of sequence.

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Ordinary Differential Equations

Unit-I

Section – A

1. Two solutions φ_1, φ_2 of L(y) = 0 are linearly independent on an interval I if and only if $\omega(\varphi_1, \varphi_2)(x) \neq 0$, for all x in I.
2. If φ_1, φ_2 are two solutions of $y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0 then show that $\omega(\varphi_1, \varphi_2)(x) = e^{-a_1(x-x_0)} \omega(\varphi_1, \varphi_2)(x_0)$.
3. State and prove existence theorem.
4. State and prove uniqueness theorem.
5. If φ_1, φ_2 are linearly independent solutions of the constant coefficient equation $y'' + a_1 y' + a_2 y = 0$ and let $\omega(\varphi_1, \varphi_2)$ be abbreviated to ω. Show that ω is a constant if and only if $a_1 = 0$.
6. Show that $\varphi_1(x) = cosx$ & $\varphi_2(x) = 3(e^{i(x)} + e^{-ix})$ are linearly dependent.

7. Solve: $y'' - y' - 2y = e^{-x}$.
8. Find all the solution φ of $y'' + y = 0$ $satisfying$ $\varphi(0) = 1, \varphi\left(\frac{\pi}{2}\right) = 2$.
9. Let φ_1, φ_2 be any two linearly independent solutions of L(y) = 0 on an interval I.

   Show that every solution φ of L(y) = 0 can be written uniquely as $\varphi = C_1\varphi_1 + C_2\varphi_2$ where C_1, C_2 are constants.
10. Find all the solution of $y'' + y = 2sinxsin2x$.
11. Find the solutions of the IVP $y'' + (4i + 1)y' + y = 0, y(0) = 0, y'(0) = 0$.
12. Show that the functions φ_1, φ_2 defined by $\varphi_1(x) = x^2, \varphi_2(x) = x|x|$ are linearly independent for $-\infty < x < \infty$.
13. Find all solutions of the equation $y'' + y = secx, \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$.
14. Find the solutions of the equation $y'' - 2y' - 3y = 0, y(0) = 0, y'(0) = 1$.
15. Find the solutions of the IVP $y'' + 2\alpha y' + (\alpha^2 + \pi^2)y = 0, y(0) = 3, y'(0) = -3\alpha$.

Section – B

16. State and Prove Existence "& Uniqueness theorem for the IVP

    $y'' + a_1y' + a_2y = 0, y(x_0) = \alpha, y'(x_0) = \beta$.
17. a) Find the solutions of the IVP $y'' + (3i - 1)y' - 3iy = 0, y(0) = 2, y'(0) = 0$.

    b) Find all the solutions of $y'' - 7y' + 6y = sinx$.
18. Prove that the two solutions φ_1, φ_2 of L(y)=0 are linearly independent on an interval I if and only if ω($\varphi_1, \varphi_2)(x) \neq 0$, for all x in I.
19. Find all the solutions of the equation $y'' - 4y' + 5y = 3e^{-x} + 2x^2$.
20. Let φ be any solution of L(y) = $y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0 , then for all x in I . Prove that $\|\varphi(x_0)\|e^{-k|x-x_0|} \leq \|\varphi(x)\| \leq \|\varphi(x_0)\|e^{k|x-x_0|}$,

    Where $\|\varphi(x)\| = [|\varphi(x)|^2 + |\varphi(x)|^2]^{1/2}, k = 1 + |a_1| + |a_2|$.
21. Let b be continuous on an interval I. Every solutions ψ of L(y) = b(x) on I can be written as $\psi = \psi_p + c_1\varphi_1 + c_2\varphi_2$ where ψ_p is a particular solution φ_1, φ_2 are two linearly independent solutions of L(y)=0 and c_1, c_2 are constants. A particular solution ψ_p is given by $\psi_p(x) = \int_{x_0}^{x} \frac{[\varphi_{1}(t)\varphi_{2}(x) - \varphi_1(x)\varphi_2(t)]}{\omega(\varphi_1,\varphi_2)(t)} dt$ conversely every such ψ is a solution of L(y)=b(x).
22. Find all solutions of $y'' - y = \frac{2}{1+e^x}$.
23. Find all solutions of $y'' - 2y' = e^x - sinx$.

Unit – II

Section - A

1. Find all the solution of $y^{(4)} - 16y = 0$.
2. Compute the Wronskian of four linearly independent solutions of $y^{(4)} + 16y = 0$.
3. Compute the Wronskian of the solution of the equations $y''' - 4y' = 0$.
4. Explain the annihilator method of solving a non- homogeneous equation solve using this method $y'' + 4y = cosx$.
5. Using annihilator method to find a particular solutions of the equation $y'' - y' - 2y = x^2 + cosx$.
6. Using annihilator method to find a particular solutions of the equation $y'' + 4y = sin2x$.

7. Consider the equation $y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = b(x)$, where $a_1, a_2, \ldots, a_n$ are real constants and b is a real – valued continuous function on some interval I. Show that any solution which satisfies real initial conditions is real – valued.
8. Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be any n constants, and let x_0 be any real number. Show that exists a solution φ of L(y) = 0 on $-\infty < x < \infty$ satisfying $\varphi(x_0) = \alpha_1, \varphi^{'(x_0)} = \alpha_2, \ldots, \varphi^{(n-1)}(x_0) = \alpha_n$.
9. Show that if f, g are two functions with k – derivatives, then
$$D^k(f,g) = \sum_{l=o}^{k} \binom{k}{l} D^{(l)}(f) D^{k-l}(g), where \binom{k}{l} = \frac{k!}{(k-l)!(l)!}.$$
10. Compute the solution ψ of $y^{'''} + y^{''} + y^{'} + y = 1\ satisfying\ \psi(0) = 0, \psi^{'}(0) = 1, \psi^{''}(0) = 0$.
11. Compute the solution φ of $y^{(4)} + 16y = 0$ satisfies $\varphi(0) = 1, \varphi'(0) = \varphi''(0) = \varphi'''(0) = 0$.
12. Find four linearly independent solutions of the equations $y^{(4)} + \lambda\, y = 0$ in case
    i) $\lambda = 0$, ii) $\lambda > 0$, iii) $\lambda < 0$.
13. Using variation of constants method, find the solution of $y^{(4)} + 16y = \cos x$.
14. Prove that if P_1, P_2, P_3, P_4 are polynomials of degree two they are linearly dependent on $-\infty < x < \infty$.
15. Find all real – valued solutions of the equation $y^{(5)} + 2y = 0$.

Section – B

16. Find the particular solution ψ_p of L(y) = b(x).
17. a) Compute the five linearly independent solution and its Wronskian of the equation
    y(5) – y(4) - y′ + y = 0.
    b) Find the solution φ of IVP $y''' + y = 0, y(0) = 0, y'(0) = 1, y''(0) = 0$.
18. Explain the annihilator method for solving the non – homogeneous equation, using this method. Find a particular solutions of the equation $y'' + y = xe^x cos2x$.
19. State & prove Existence & uniqueness theorem.
20. Consider the equation with constant coefficients L(y)= p(x) e^{ax}, where p is the polynomial given by $p(x) = b_0 x^m + b_1 x^{m-1} + \cdots + b_m, (b_0 \neq 0)$. Suppose a is a root of the characteristic polynomial p of L of multiplicity j, then there is a unique solution ψ of L(y)= p(x) e^{ax} of the form $\psi(x) = x^j(c_0 x^{m-1} + \cdots + c_m)e^{ax}, where\ c_0, c_1, \ldots, c_m$ are constants determined by the annihilator method.

Unit – III

Section – A

1. Prove that there exist n linearly independent solutions of L(y)= 0 on I.
2. Using the fact that $P_0(x)=1$ is a solution of $(1 - x^2)y'' - 2xy' = 0$
   find a second independent solution.
3. Verify that $\varphi_1(x) = x$ is the one solution of $(1 - x^2)y'' - 2xy' + 2y = 0$ and find a second independent solution.
4. Show that $p_n(-x) = (-1)^n p_n(x)$ and hence that $p_n(-1) = (-1)^n$.
5. Find two linearly independent power series solution for $y'' + y = 0$.

6. If one solution of $xy'' - (x+1)y' + y = 0 \; is \; \varphi_1(x) = e^x (where \; x > 0)$ then find a second independent solutions.
7. Find two linearly independent power series solutions of $y'' + x^3y' + x^2y = 0$.
8. Show that the coefficients of $x^n \; in \; p_n(x) \; is \; \frac{(2n)!}{2^n(n!)^2}$.
9. One solution of $x^3y'' - 3x^2y'' + 6xy' - 6y = 0$ for x > 0 is $\varphi_1(x) = x$. Find a basis for the solutions for x > 0.
10. Two solutions of $x^3y'' - 3xy' + 3y = 0, (x > 0) are \; \varphi_1(x) = x, \varphi_2(x) = x^3$. use this information to find a third independent solution.
11. Find all solutions of the equation $y'' - \frac{2}{x^2}y = x, (0 < x < \infty)$.
12. Prove that $\int_{-1}^{1} xp_n(x)p_{n-1}(x)dx = \frac{2n}{4n^2-1}$.
13. Let $\varphi_1, \varphi_2, \dots, \varphi_n$ be n solutions of $y^{(n)} + a_1y^{(n-1)} + \cdots + a_ny = 0$ on an interval I and let x_0 be any point in I. Then show that $\omega(\varphi_1, \varphi_2, \dots, \varphi_n)(x) = \exp[-\int_{x_0}^{x} a_1(\mathrm{t})\mathrm{dt}] \; \omega(\varphi_1, \varphi_2, \dots, \varphi_n)(x_0)$.
14. Find two linearly independent solution of the equation
$$(3x-1)^2y'' + (9x-3)y' - 9y = 0 \; for \; x > \frac{1}{3}$$
15. Show that $p_n(x)$ is the coefficient of z^n in the expansion of $(1 - 2xz + z^2)^{-\frac{1}{2}}$ in ascending powers of z. ie., $(1 - 2xz + z^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n p_n(x), \; |x| \leqq 1, |z| < 1$.

Section – B

16. Find the solution of $y'' + (x-1)^2y' - (x-1)y = 0$ in the form $\phi(x) = \sum_{k=0}^{\infty} C_k(x-1)^k$, which satisfies $\phi(1) = 1, \phi'(1) = 0$.
17. Find the solution $\phi$of $xy'' + y' + 2y = 0$ in the form $\phi(x) = \sum_{k=0}^{\infty} C_k(x-1)^k$, Satisfying $\phi(1) = 1, \phi'(1) = 2$.
18. Find the particular solution $\psi_p \; of \; L(y) = b(x)$.
19. Find two linearly independent power series solution of the equation $y'' - xy' + y = 0$.
20. Solve the Legendre equation.

UNIT- IV

Section - A

1. Compute the indicial polynomial and their roots for the equation $x^2y'' + (x - 3x^2)y' + e^xy = 0$.
2. Compute the indicial polynomial and the roots of the equation $x^2y'' + (x + x^2)y' - y = 0$.
3. Compute the indicial polynomial and the roots of the equation $x^2y'' + xe^xy' + y = 0$.
4. If λ , μ are positive show that $(\lambda^2 - \mu^2) \int_{-1}^{1} \phi_\lambda(x)\phi_\mu(x)dx = \phi_\lambda(1)\phi'_\mu(1) - \phi'_\lambda(1)\phi_\mu(1)$

5. Show that $x^{\frac{1}{2}}J_{\frac{1}{2}}(x) = \frac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} sinx.$

6. Locate and classify the singular points $(x^2 + x - 2)^2 y'' + 3(x + 2)y' + (x - 1)y = 0.$

7. Prove that the series k_0 converges for $|x| < \infty$.

8. Show that $J'_0(x) = -J_1(x)$.

9. Show that $J_{\alpha-1}(x) - J_{\alpha+1}(x) = 2J'_\alpha(x)$.

10. Find all solutions of the equations for x > 0 $x^2y'' + 2xy' - 6y = 0.$

11. Find all the solutions ϕ of the form $\phi(x) = |x|^2 \sum_{k=0}^{\infty} C_k x^k, (|x| > 0)$. For the equation

    $3\,x^2y'' + 5xy' + 3xy = 0$

Section – B

12. Obtain two linearly independent solutions of the equations $x^2y'' - 2x^2y' + (4x - 2)y = 0.$
13. Derive Bessel's function of zero order of find kind, J_0.
14. Find all solutions of $x^3y''' + 2x^2y'' - xy' + y = 0$ for x > 0.
15. Derive Bessel's function of order α of find kind.
16. Obtain the general case of second order equation with regular singular points.

UNIT – V

Section – A

1. Find the solution of $y' = 2y^{1/2}$ passing through the point (x_0, y_0) $where$ $y_0 > 0.$
2. Show that the functions $f(x, y) = x^2cos^2y + ysin^2x$ on $S: |x| \leq 1, |y| < \infty$ satisfy that Lipchitz conditions on the set S.
3. Verify that the equation *(x + y)dx + (x – y)dy = 0* is exact and solve it.
4. Consider the equation $y' = x^2 + y^2, y(0) = 0$ on $R: |x| \leq 1, |y| \leq 1.$
   Compute the upper bound M for the functions $f(x, y) = x^2 + y^2$ on R.
5. Find the real valued solution of the equation $y' = \frac{x+x^2}{y+y^2}$.
6. Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on $I.$
7. Find an integrating factor of $(e^y + xe^y)dx + xe^ydy = 0$ and hence solve it.
8. Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ of $y' = x^2 + y^2$,y(0) = 0.
9. Solve *(x + y)dx + (x - y)dy = 0.*
10. Solve $y' = \frac{x+y+1}{2x+2y-1}$.
11. Find all real – valued solutions of the equation $y' = x^2y$.
12. Write down the method of successive approximation.

Section – B

13. (a) Find all the real valued solution of the equation $y' = \frac{x^2+xy+y^2}{x^2}$.

    (b) Find the sequence of successive approximations for the problem $y'' = x - y, y(0) = 1, y'(0) = 0$

and show that the sequence tends to a limit for all real x.

14. Prove that the existence theorem for convergence of the successive approximation.
15. a) Let M,N be two real valued function which have continuous first order partial derivatives on some
 rectangle $R: |x - x_0| \leq a, |y - y_0| \leq b$. Then prove that the equation $M(x,y) + N(x,)y' = 0$ is
 exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

 b) Solve *(x + y)dx + (x - y)dy = 0.*

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ELECTIVE - GRAPH THEORY

6-MARK QUESTIONS

UNIT-I-GRAPHS, SUBGRAPHS AND TREES

1. Prove that $\sum_{v=V} d(v) = 2 \in$ and deduce that in any graph, the number of vertices of odd degree is even.
2. Let G be bipartite, Show that the vertices of G can be enumerated so that the adjacency matrix of G has the form $\begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$ whereA_{21} is the transpose of A_{12}.
3. Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.
4. If G is a tree, then show that$\in = V - 1$.
5. Prove that a connected graph is a tree if and only if every edge is a cut edge.
6. If G is a k-regular bipartite graph with k ≥ 0 has bipartition (X,Y), then Show that |X|=|Y|.
7. Define Incidence and adjacency matrix give example.
8. In any graph the number of vertices of odd degree is even.
9. Every non trivial tree has atleast two vertices of degree one.
10. Show that every connected graph contains a spanning tree.

UNIT-II-CONNECTIVITY, EULER AND HAMILTINION CYCLE

11. If G is a block with $\gamma \geq 3$ prove that any two edges of G lie on a common cycle.
12. Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.
13. Show that K≤ $K' \leq \delta$.
14. Let G be simple graph and let u and v be non adjacent vertices in G such that d(u)+d(v)≥ γ then G is Hamiltonion iff G+uv is Hamiltonion.
15. Show that the Hershel graph is non- Hamiltonion.
16. If G is Hamitonion then for every non-empty subset s of V, $\omega(G - S) \leq |S|$.
17. If G is non Hamiltonion simple graph with $\gamma \geq 3$ then is degree majorised by some $C_{m,\gamma}$.
18. Prove that C(G) is well defined.
19. If G is two connected then any two vertices of G are lie on a common cycle.
20. A non empty connected graph is Eulerian iff it has no vertices of odd degree.

UNIT-III-MATCHING, EDGE COLOURING

21. Let G be a connected graph that is not an odd cycle. Show that G has a 2-edge coloring in which both column are represented at each vertices of degree at least two.
22. Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

23. If G is a K- Regular bipartite graph with K>0, then prove that G has a perfect matching.
24. Let g be a connected graph that is not an odd cycle. Then prove that G has a 2- edge coloring in which both colors are represented at each vertex of degree at least two.
25. Let M be a matching and be a covering such that |M|=|K| then M is a maximum matching and is minimum covering.
26. State and prove Konig's theoremIf G is bipartite then $\varkappa' = \Delta$

UNIT-IV-INDEPENDENT SETAND CLIQUES, VERTEX COVERING

27. If $\delta > 0$, with usual notation prove that $\alpha' + \beta' = \gamma$.
28. If G is simple, prove that $\pi_k(G) = \pi_K(G - e) - \pi_{K(G.e)}$ for any edge e of G.
29. Using the standard notations, Show that $\alpha + \beta = \gamma$
30. If G is K-Critical, than show that $\delta \geq k - 1$.
31. State and prove Gallai's Theorem.
32. Every k-cromatic graph has atleast k-vertices of degree atleast k-1.
33. Show thatb in a critical graph no vertex cut is clique.
34. Prove that r(2,2)=2
35. Prove that R(3,3)=6
36. Prove that R(3,4)=9

UNIT-V-PLANAR GRAPHS

37. Prove that a graph G is embeddable in the plane if and only if is embeddable on the sphere.
38. If G is a simple planar graph with v≥3 vertices, then prove that $\in \leq 3v - 6$, and hence show that K_5 is nonplanar.
39. State and prove Euler's formula for connected plane graoh.
40. Show that every Hamiltonion plane graph is 4-face colorable.
41. Prove that $K_{3,3}$ is non planar.
42. Show that if G is a simple plansr graph then $\delta \leq 5$
43. Show that if G is a plane graph then $\sum_{f \in F(G)} d(G) = 2 \in$
44. Show that all planar embedding of a given connected planar graph have the same number of faces.

15- MARK QUESTIONS

UNIT-I

1. Prove that A graph is bipartite iff it contain no odd cycle.

2. Define bipartite graph and give an example. Show that a graph is bipartite iff it contains no odd cycle.
3. Let T be a spanning tree of a connected graph G and let e be an edge of G not in T then T+e contains a unique.
4. A vertex V of a tree G is a cut vertex of G iff d(v)>1.
5. If e is a link of G then $\tau(G) = \tau(G - e) + \tau(G.e)$

UNIT-II

6. State and prove Dirac's Theorem for Hamiltonion graph.
7. State and prove Whiteney's theorem.
8. State and prove Chavatal's theorem.
9. Show that if has no vertices of odd degree then there are edge disjoint cycleC_1,C_2, C_3….C_m such that E(G)=E(C_1)UE(C_2)U…UE(C_m).
10. Show that if G has 2k>0 vertices of odd degree then there are edges disjoint trail Q Q….Q inG such that E(G)=E(Q_1)UE(Q_2)U....UE(Q_k)

UNIT-III

11. State and prove vizing,s theorem.
12. State and prove Berge theorem.
13. State and prove Hall's theorem.
14. Let G be a connected graph is not an odd cycle then G has 2-edge coloring in which both colors are represented at each vertex of degree atleast 2.
15. Let $\varrho = (E1, E2, E3 \dots E$k) be a optimal k edge coloring of G. If there is a vertex u in G and colors i and j such that I is not represented at u and j is represented atleast twice at u then the component of G [$E_i \cap E_j$] that contains u is an odd cycle.

UNIT-IV

16. Find chromatic polynomial for the graph $K_{1,3}$ and C_4.
17. State and prove brooks theorem.
18. Show that G be a k- critical graph with a two vertex cut u,v then d(u)+d(v)$\geq 3k - 1$
19. State and prove Dirac theorem.
20. Show that for any graph G $\pi_k(G)$ is a polynomial in k of degree γ with a integer coefficient leading term k^γ and constant term zero. Furthermore the coefficient of $\pi_k(G)$ alternate inside.

UNIT-V

21. State and prove Five color theorem.
22. Prove that every planar graph is 5- Vertex colorable.

23. Show that the following three statement are equivalen
    (i)Every planar graph is 4-vertex colorable
    (ii)Every plane graph is 4-face colorable
    (iii)Every simple 2-edges connected 3-regular planar graph is 3-edge colorable.
24. Show that if G is connected plane graph then $\gamma - \epsilon + \varphi = 2$

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OPEN ELECTIVE - INTRODUCTION TO COMPUTER APPLICATIONS

Two Mark Questions:

1. Define computer.

2. Define data.

3. Define information.

4. Define data processing.

5. List out the generation of computer.

6. Define personal computer.

7. Define error detecting code.

8. What is meant by memory?

9. Define RAM.

10. Define ROM.

11. Define secondary memory.

12. Define CPU.

13. Define CDROM.

14. Define network.

15. List some output devices.

16. Write the types of networks.

17. Define LAN.

18. Define WAN.

19. What is meant by flat panel display?

20. Define software.

21. Define database.

22. Define operating system.

23. Define WWW.

24. Define E-mail.

25. Define web browser.

26. write the different parts of excel window.

27. How many rows and columns are in excel spreadsheet.

28. Specify any four types of charts in excel?

29. How will you save a file in MS Excel?

30. What is a worksheet?

31. Write any two mathematical functions in excel.

32. How to apply international currency style using MS Excel?

33. How would you start ms excel?

34. How do you insert the new columns in MS Excel?

35.Write the different part of word window.

36. How will you save a file in MS -word?

37. What is mail merge?

38. What is the command for insert clip art?

39. What is auto correct options.

40. How can we numbering the list in MS-word?

41. What is windows?

42. Define icon

43. What is GUI?

44. How to create a new folder in windows.

45. How to rename a folder in windows.

46. How to copy a folder in windows.

47. Define file management.

48. Define files and folders.

49. What is windows explorer?

50. Define operating system.

51. Define multitasking.

52. Define multithreading.

53. Define desktop operating system.

54. List out any two difference between windows and linux.

55. Define disk operating system.

56. What are the types of DOS?

57. Define windows.

58. List out any four internal commands in DOS.

59. List out any four external commands in DOS.

60. What are the features of windows 8?

Five Mark Questions:

1. Explain in detail about the desktop computer.

2. Explain the categories of computer.

3. Explain about input units.

4. Explain about the personal computer.

5. Explain about characteristics of computer.

6. Explain about primary and secondary memory.

7. Explain about the video display devices.

8. Explain about the specification of CPU.

9. Write the application of LAN.

10. Write short note on future of internet technology.

11. Explain about the structure of database.

12. Explain about the data organization.

13. Write the characteristics of database management system.

14. Explain the history WWW.

15. Explain the history of internet.

16. Discuss about Web Browsers and its categories.

17. List out and explain the features of internet explorer.

18. Explain about the Browser.

19. How to search in the internet?

20. Write the uses of E- mail.

21.Write short notes on control panel.

22.Explain structure of windows.

23.Discuss about elements of windows in detail.

24.Explain the importance of file management.

25.How will you create,copy,move and delete a folder?Explain.

26.Discuss internet explorer security.

27.Explain manipulating windows.

28.Discuss about evolution of operating systems.

29.Explain the functions of operating system.

30.Explain the classification of operating system.

31.Discuss the examples of operating system.

32.Explain about disk operating system.

33.Difference between windows and DOS.

34.Difference between linux and windows.

35.Discuss windows families.

36.Explain the features of windows7.

Ten Mark Questions:

1. Discuss about generation of computer.

2. Write about the classification of computer.

3. Explain about the memory cell

4. Detailed about the structure of CPU.

5. Explain about the types of networks.

6. Detailed about output devices.

7. Discuss about Programming Language and its types.

8. Explain about the operating System.

9. How to create an E- mail ID? Explain with example.

10. What is meant by Web browsers? Explain.

11.What are the various types of charts that are available in MS-excel?explain.

12.What are the steps for copying and pasting formula in MS excel.

13.Explain about the creation of charts in MS excel.

14.Explain how to print excel document with its options.

15.Explain the procedure to create charts in excel

16.Craete a excel worksheet containing S.No. product code, product name,price of the product,revenue through the product.fill in the necessary information and sort the data revenue wise.

17.Describe how would you insert a chart in the worksheet.

18.Explain the functions in excel.

19.Explain the mail merge in detail with an example.

20.Explain the method of creating a new document in word.

21.Write a short note on find and replace facility.

22 Discuss the procedure to use bullets and numbering.

23. Write the steps in design your bio- data using with MS-word.

24.Explain how to print word document with its options.

25.Explain how to inserting picture in to your document.

26.Discuss how to creating a template in detail.

27.Explain the features of MS- word.

28.Explain the various components of MS-word document.

29.Explain about features of windows XP

30.Discuss about windows explorer.

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II M.Sc., MATHEMATICS

SEMESTER III

COMPLEX ANALYSIS I - DMA 31

UNIT-1: ANALYTIC FUNCTION AND POWER SERIES

SECTION A – 2 MARKS

1. Define differentiable
2. Define entire function
3. Define analytic function
4. Define harmonic function
5. Define simply connected
6. State anti derivative theorem
7. Define root test
8. Define ratio test
9. Define periodic
10. Define holomorphic branch.

SECTION B – 5 MARKS

1. A real- valued function of a complex variable either has derivative zero or the derivative does not exist.
2. If f and g are differentable at Z_0 , then their sum of f+g difference f-g product fg, quotient $\frac{f}{g}$ and the sclar multiplication cf, are also differentiable at Z_0 and $(f \pm g)' = f'+ g'$, $(fg)' = f'$ g+ f g', $(\frac{f}{g})' = \frac{f' \text{ g}- \text{f } g'}{f^2}$,$(cf)' = Cf'$.
3. If f(z)= u(x,y)+iv(x,y) is differentiable at Z_0, then the C-Requation hold at $Z_0 = x_0$ +iy_0 , i$f_x(Z_0) = f_y(Z_0)$ or equivalently $u_x(x_0, y_0) = v_x(x_0, y_0)$ and $u_x(x_0, y_0)$= $-v_x(x_0, y_0)$.
4. Let u:$\Omega \subseteq R^2 \rightarrow R$ where Ω is an open set containing the point (x_0, y_0) suppose that
   (i) $u_x$and$v_x$exist at a every point in a neighborhood of (x_0, y_0) and
   (ii) $u_x$andu_y are continuous at (x_0, y_0).
   Then , for sufficiently small s and t in R
   $u_x(x_0 + s, y_0 + t)$ - u(x_0, y_0) = $su_x(x_0 + s^*, y_0 + t)$ $+ tu_y(x_0, y_0 + s^*)$ with $|s^*| < |s|$ and $|t^*| < |t|$
5. Let Ω be an open subset of C. Then the real and imaginary parts of an analytic function in Ω are harmonic in Ω
6. If u(x,y) = 4xy-x^3+3xy^2, then u_x= 4x-3x^2+3y^2 and u_y= 4x+6xy.
7. Let Ω = {(x,y)∈ R^2:$|x - a| < \delta$,$|y - a| < \delta'$} be an open rectangle in R^2. Suppose that f,g∈ $C'(\Omega)$ such that f_y=g_x in Ω then their exist a function v∈ $C^2(\Omega)$ satisfying the conditions v_x= f and v_y=g in Ω.
8. State and prove Antiderivative theorem.
9. State and prove Ratio test.
10. A power series $\sum_{n\geq 0} a_n Z^n$ and k-times derived series defined series defined by $\sum_{n\geq k} n(n-1) \dots \dots (n-k+1)a_n Z^{n-k}$ have the same radius of convergence.
11. Suppose f(z)= $\sum_{n\geq 0} a_n Z^n$ and g(z)= $\sum_{n\geq 0} b_n Z^n$ converge for $|z| < R_1$ and $|z| < R_2$ respectively . If f(z_k)= g(z_k) for a sequence $\{z_k\}$ of nonzero complex numbers in 0< $|z| < \delta$ such that $z_k \rightarrow 0$ as k→ ∞, if f(z)=g(z) for all z with $|z| < \delta$, where $0 < \delta <$ min$\{R_1 R_1\}$, then a_n=b_n for every n∈ N_0.
12. Let f:D $\rightarrow C$ be an analytic domain in D not containing 0. Then is a branch of log z in D iff f'(z)=$\frac{1}{z}$ for all z∈D and exp {f(a)}=a for at least one a∈D.
13. In the unit disc Δ, the power series $\sum_{n\geq 1} \frac{z^n}{n}$ represent the logarithmic function – Log(1-z).

SECTION C – 10 MARKS

1. Let f:$D_1 \rightarrow C$, g:$D_2 \rightarrow C$ be such that f(D_1) ⊆ D_2 . If f is differentiable at Z_0 and g is differentiable at w_0=f(Z_0), then the composition (gof)(z)=g(f(z)) is differentiable atZ_0 and (gof)'(Z_0)=g'(w_0) f'(Z_0)=(g'of)(Z_0) f'(Z_0).
2. Let f:$\Omega \rightarrow C$ where Ω is an open set in C containing the point Z_0 , and f(z)=u(x,y)+iv(x,y) for z= x+iy ∈ Ω. Suppose that

(i) $u_x u_y v_x$ and v_y exist at each point in a neighborhood of Z_0 and are continuous at Z_0

(ii) $The\ C - R$ equations are valid at Z_0

3. Suppose that f is analytic in a domain. we have
   (i) If f'(z)= 0 in D , then f is constant
   (ii) Iff$|f|$,Re f, Im f, Arg f is constant
4. Polar form of the C-R equation and laplacian.
5. State and prove Root test
6. If $\sum_{n\geq 0} a_n Z^n$ has radius of convergence R>0, then f(z)=$\sum_{n\geq 0} a_n Z^n$ is analytic in $|z| <$ R, f^k(z) exist for every k∈N and f^k(z)=k! a_k+$\sum_{n\geq k+1} \frac{n!}{n-k} a_n Z^{n-k}$ where a_k=$\frac{f^k(0)}{k}$.
7. Explain binominal series.

UNIT II - COMPLEX INTEGRATION
SECTION – A (2 marks)

1. Define simple closed curve.
2. Define continuously differentiable.
3. Define piecewise smooth curve.
4. Define contour integral.
5. Define reparameterization.
6. Define length of curve.
7. Evaluate $\int_\gamma z^n dz$.
8. Evaluate $\int_\gamma xdz, if\ \gamma$is the line from 0 to 2i and then from 2i to 4+2i.
9. Evaluate $\int_\gamma (z^2 + z)dz$.
10. State Cauchy - Goursat theorem.
11. State Cauchy's Integral Theorem.
12. Define annulus.
13. Define winding number.
14. State Cauchy Integral formula.
15. State Gauss' Mean Value theorem.
16. Evaluate $\int_{|z|=r} Rez\ dz$.
17. State Weierstrass' theorem for series
18. State Taylor's theorem.
19. Define Laurent series.
20. State Laurent's theorem.
21. Compute $\int_\gamma |z|^2 dz, where\ \gamma(t) = t + it^3, 0 \leq t \leq 1$.
22. Compute $\int_\gamma |z|^2 dz, where\ \gamma(t) = it, 0 \leq t \leq 1$.
23. Compute $\int_\gamma |z|^2 dz, where\ \gamma(t) = acost + ibsint, a, b \in \mathbb{R},\ 0 \leq t \leq 2\pi$.
24. Use Cauchy integral formula, to evaluate $\int_{|z-1|=1} (\overline{z})^n dz$.
25. Use Cauchy integral formula, to evaluate $\int_{|z-a|=1} \frac{e^{2\pi z}}{z-a} dz$.

SECTION – B (5 marks)

26. Evaluate $I_j = \int_{\gamma_j} xdz, j = 1\ to\ 4, where$
    i) γ_1 is the straight line segment from 0 to $a + ib$, $(a, b \in \mathbb{R})$
    ii) γ_2 is the circle $|z| = R$.
    iii) γ_3 is the line segment from 0 to 1, 1 to 1+i and then from 1+i to 0.
    iv) γ_4 is given by $\gamma_4(t) = t + it^2\ on\ [0,1]$.
27. Prove that $\int_\gamma f(z)dz = -\int_{-\gamma} f(z)dz$.
28. If L= L(γ) is the length of the curve and $M = \max_{t\in[a,b]} |f(\gamma(t))|, then\ \left|\int_\gamma f(z)dz\right| \leq ML$.
29. State and prove Interchange of limit and integration.
30. Prove that $\int_\Gamma f(z)dz = \int_\gamma f(z)dz$.
31. Evaluate $\int_\gamma z^2 dz\ where\ \gamma(t) = 1 - tsin\pi t + i(t + cos\pi t)$ which is an arc connecting 1+i to 0.
32. Evaluate $\int_\gamma \frac{dz}{z}\ where\ \gamma$ is an arc joining 1-I to 1+i.
33. If f is analytic with f' continuous inside and on a simple closed contour γ, then $\int_\gamma f(z)dz = 0$.
34. If D is an open set and let f be analytic on D except possibly at $a \in D$. Assume that f is continuous on D. Then, we have $\int_{\partial T} f(z)dz = 0$ for every closed triangle T in D.
35. If D is a domain that is starlike with respect to a and f is analytic on D. Then there exists an analytic function F on D such that $F'(z) = f(z) in\ D. In\ particular\ \int_C f(z)dz = 0$ for every closed contour C in D.
36. If Ω is a simply connected domain and $f \in H(\Omega)$ with $f(z) \neq 0$ on Ω. Then there exists a $h \in H(\Omega) such\ that\ e^{h(z)} = f(z)$.
37. If Ω is a simply connected domain and $f \in H(\Omega)$ with $f(z) \neq 0$ on Ω. Then f has an analytic square root- that is there exists a g$\in H(\Omega) with\ g^2 = f(z)\ for\ z \in \Omega$.
38. If $f \in H(\mathbb{C})$ with $f(z) \neq 0$ on C. Then, there exists a $h \in H(\mathbb{C})$ such that $f(z) = e^{h(z)}$.
39. For every closed contour γ in $\mathbb{C}$ and $a \in \mathbb{C}/\gamma, n(\gamma; a)$is an integer.
40. If γ is a closed contour in $\mathbb{C}$, then the mapping $\zeta \to n(\gamma; \zeta)$ is a continuous function of ζ at any point $\zeta \notin \gamma$.
41. If D is a simply connected domain and γ is a closed contour in D, then for

$$f \in \mathcal{H}(D) and\ a \in D/\{\gamma\}, \qquad f(a)n(\gamma; a) = \frac{1}{2\pi i}\int_\gamma \frac{f(\zeta)}{\zeta - a} d\zeta.$$

42. State and prove Gauss's Mean – Value Theorem.
43. State and Prove Cauchy Integral formula for derivatives.
44. If $f = u + iv$ is analytic in a domain D, then all the partial derivatives of u & v exist and are continuous in D.
45. If f is analytic in the open disk $\Delta(a; R) and\ |f(\zeta)| \leq M\ for\ \zeta \in \partial\Delta(a; r), 0 < r < R$. Then for each k ∈N onehas $\left|f^{(k)}(a)\right| \leq Mk!\, r^{-k}$.
46. If $(z) = \log z\ for\ z \in \mathbb{C}\{x + i0: x \leq 0\}\ and\ a = -1 + i = \sqrt{2}e^{3\pi i/4}$. Then

$$f(a) = In\sqrt{2} + \frac{3\pi i}{4} and\ for\ n \geq 1.$$

47. A function f analytic at a has a zero of order m at a iff$f(z) = (z-a)^m g(z)$, where g is analytic at a and g(a) $\neq$ 0.
48. Every zero of an analytic function $f(\not\equiv 0)$ is isolated.
49. If the power series $\sum_{n\geq 0} a_n z^n$ and $\sum_{n\geq 0} b_n z^n$ are convergent for $|z| < R_1 and |z| < R_2$

with sums $f(z)$ and $g(z)$, respectively. Then we have $f(z)g(z) = \sum_{n\geq 0} C_n z^n$; $C_n = \sum_{k=0}^{n} a_k b_{n-k}$,for$|z| < \min\{R_1, R_2\} = R$.

50.State and prove Cauchy inequality.

SECTION – C (10 marks)

51. If $f = u + iv$ is analytic in an open set D, co0ntaining a contour γ with parametric interval [a,b]. ie.,$\gamma([a,b]) \subset D\ then\ \int_\gamma f'(z)dz = f(\gamma(b)) - f(\gamma(a))$. That is, the value of the integral is independent of the path. In particular, we have $\int_\gamma f'^{(z)}dz = 0, if\ \gamma is\ closed.$

52. State and Prove Cauchy-Goursat Theorem.

53. State and prove Cauchy's Integral Theorem.

54. If ϕ is a complex – valued function which is continuous in an open set D containing a contour γ. Then for all $z \notin \gamma, the\ function\ F_n defined\ by\ F_n(z) = \int_\gamma \frac{\phi(\zeta)}{(\zeta - z)^n} d\zeta, n = 1,2,3 \ldots$ is analytic and satisfies the equation $F_n'^{(z)} = nF_{n+1}(z)$; or equivalently$F^{(k)}(z) = k!\int_\gamma \frac{\phi(\zeta)}{(\zeta-z)^{k+1}} d\zeta$, $k = 1,2 \ldots with\ F_1(z) = F(z)$.

55. State and prove Weierstrass' theorem for sequences.

56. If f is analytic in Δ_R, then f has a Maclaurin series expansion $f(Z) = \sum_{n\geq 0} a_n z^n\ \ for\ all\ z\epsilon\Delta_R$, where$a_n = \frac{f^n(0)}{n!} = \frac{1}{2\pi i}\int_\gamma \frac{f(\zeta)}{\zeta^{n+1}} d\zeta, n = 0,1,2 \ldots with\ \gamma = \{\zeta : |\zeta| = r\} and\ 0 < r < R.$

57. If f is analytic in a domain D,then for $z\epsilon\Delta(a;R) \subseteq D, f$ has the Taylor series expansion f$(Z) = \sum_{n\geq 0} a_n (z-a)^n a_n = \frac{f^n(a)}{n!} = \frac{1}{2\pi i}\int_\gamma \frac{f(\zeta)}{(\zeta-a)^{n+1}} d\zeta\ \ where$ C= $\{\zeta : |\zeta - a| = r\} and\ 0 < r < R.$

58. If f is analytic is an domain D. If S, the set of zeros of f in D, has a limit point z^*in D, then f(z)≡0 in D.

59. If f is analytic in the annulus $R_1 < |z| < R_2$, then f has a unique representation Maclaurin series expansion$f(Z) = \sum_{n\geq 0} a_n z^n\ \ for\ any\ zin\ tha\ annulus$ where $a_n = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{\zeta^{n+1}} d\zeta, n\epsilon\mathbb{Z}$

$$with\ C = \{\zeta : |\zeta| = r\} and\ R_1 < r < R_2.$$

60. Evaluate the integral $I = \int_\gamma \frac{Arctanz}{z^2 - 1\ dz,}$ where γ is closed contour.

UNIT :3CONFORMAL MAPPINGS AND MOBIUS TRANSFORMATIONS

SECTION A – 2 MARKS

1.Define Principal of conformal mappings.

2. Define Isogonal .

3. Define critical point with example.

4. Define rotation.

4. Define Mobius maps.

5. Define matrix interpretation.

6. Define fixed point.

7. Define cross ratio.

8. Define invariant under Mobius maps.

9. Define triple to triples under Mobius maps.

10.Conformalself maps of unit disk.

SECTION B – 5 MARKS

1.Let fε H(Ω). Where Ω is a domain containing a smooth curve Υ(t) ε t[0,1] passing through a point Z_0 ε Ω and $f'(z_0)$≠0.Then the tangent to the curve Γ: $\Gamma\ (t) = f(z) = (fo\Upsilon)(t), t$ Ɛ[0,1] at w_0=f(z) is $(fo\Upsilon)'(t_0)$=$f'(z_0)\ \Upsilon'(t_0)$.

2. Let fε H(Ω) and z_0 Ɛ Ω such that $f'(z_0)$≠0.Then f is conformal at z_0.

3.Howconformality may fail at a point z_0.where $f'(z_0)$=0.

4.Suppose f is an analytic at z_0 and $f'(z)$ has a zero of order n-1to z_0. If two smooth curves intersect at an angle αin the z place .Then their image intersect at an angle nα in the place.

5.To find the image of harizondal and vertical line segments in
Ω={z ƐC/|Re Z| $\leq \frac{\pi}{2}$.

6.Write basic properties of Mobius maps with examples.

7.Mobius transformation is generated by four types translation ,rotation, magnification and an invention.

8.The inverse of a Mobius transformation is also a Mobius transformation.

9.Explain the image of circles and lines under Mobius maps.

10. every Mobius transformation maps circles in c_∞ onto circles in c_∞ .

11.If S and T are two Mobius transformation .which agree at three distinct points of c_∞ .

12.Every Mobius transformation which lines 0,1, ∞ is necessarily the identity map.

SECTION C – 10 MARKS

1.State and Prove circle preserving property.

2.Under the function w=$\frac{1}{z}$ we have ,

i) The image of a line through the origin is a line through the origin.

ii) The image of a line not through the origin is a line through the origin.

iii) The image of a circle through the origin is a line not through the origin.

iv) The image of a circle not through the origin is a circle not through the origin.

3.Every Mobius transformation T: $c_\infty \rightarrow c_\infty$ unless T(z) =Z .

4.Ifz_0 is the consident fixed point in C of w= S(z) .Then $\frac{1}{S(z)-z_0}$= $\frac{1}{z-z_0}$+ k

5.Every Mobius transformation S(z0 which has exactly two distinct fixed point z_1 and z_2 in c_∞ can be written as $\begin{cases} \frac{S(z)-z_1}{S(z)-z_2} = \alpha\left(\frac{z-z_1}{z-z_2}\right) if\ z_1 \text{ and } z_2 \in C \\ s(z) - z_1 = \alpha(z - z_1) if\ z_{2=\infty} \end{cases}$

6.Given three distinct points z_1, $z_2 and\ z_3$ in c_∞ their exists a unique Mobius transformation T(z) such that T(z) =0,T(z_1)=1 and T(z_3)=∞.

7.The cross ratio is invariant under Mobius transformations.

8.The four distinct points $z, z_1z_2z_3$ in c_∞all on a circle or a line iff their cross ratio $(z, z_1z_2z_3)$ is a real number.

9.Find their Mobius transformation which sends 0 to 1 , i to 0 and ∞ to 1.

10.If $\{z, z_1, z_2, z_3\}$ and $\{ w, w_1, w_2, w_3\}$ are two sets of triples of distinct points in c_∞ .Then their exist a unique Mobius transformation taking z_j to w_j and it is given by, $\frac{(w-w_2)(w_{2-w_3})}{(z-z_3)(z_2-z_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$

UNIT-4: ANALYTIC FUNCTION AND POWER SERIES

SECTION A – 2 MARKS

1. State maximum modulus principle.
2. State minimum modulus principle.
3. State Hadamar’s three circles theorem
4. State Hadamar’s line circles theorem
5. State schwar’z lemma
6. State schwar’z –pick lemma

SECTION B – 5 MARKS

1. State and prove maximum modulus principle.
2. State and prove minimum modulus principle
3. If f is analytic and satisfies $|f(z)| < M$ in $\Delta(a, R)$ and f(a)=0, then
   (i) $|f(z)| < M|z - a|$ /R for every z $\in \Delta(a, R)$

   (ii) $|f'(z)| < M$ /R with the sign of equality iff f has the form

   f(z)= M∈ $(z - a)$/R for some constant ∈ with |∈|=1.

4. Every univalent analytic function f from Δ onto itself that has an analytic inverse must be of the form f(z)= $e^{i\theta}\frac{z-z_0}{1-z\overline{z_0}}$ where z_0 is a complex number $|z_0|$ <1 and 0< θ <2 π
5. State first proof of Liouille's theorem
6. State second proof of Liouille's theorem
7. State and prove generalized version of liouville's theorem
8. State and prove Gauss's theorem

SECTION C – 10 MARKS

1. State and prove maximum minimum principle theorem for harmonic functions
2. State and prove phragmen –lindelof theorem
3. State and prove harmard's thee lines theorem
4. State and prove harmard's thee circles theorem
5. State and prove schwarz lemma
6. State and prove Schwarz-pick lemma
7. Let f(z)= $\sum_{n\geq 0} a_k Z^k$ be a non-constant polynominal of degree n≥ 1 with complex coefficients. Then p has n zeros in c. That is, there exist n complex numbers $z_1, z_2, \ldots \ldots \ldots z_n$, not necessarily distinct, such that p(z)=$a_n \prod_{k=1}^{n} z - z_k$.

UNIT : 5 CLASSIFICATION OF SINGULARITIES

SECTION A – 2 MARKS

1. Define isolated singularities.
2. State Picard' little theorem
3. Define non isolated singularities.
4. Define Removable singularities.
5. Define pole.
6. Define essential singularities
7. State Picard' theorem.
8. Define Meromorphic functions.
9. State Laurent's series.
10. Define isolated singularities at infinity.

SECTION B – 5 MARKS

1.If f(z) has an isolated singularities at z_0.Then f(z) has a pole at z_0 iff lim f(z)=∞.

2. If f(z) has an isolated singularities at z_0.Then f(z) has an essential singularities atz_0 iff lim f(z) fails to exist either as a finite value or as an infinite limit.

3.An isolated singularities at z_0 of f(z) is a pole of order n iff f(z)=$(z-z_0)^{-n}$g(z),where g is an analytic at z_0 and g(z_0)≠0.

4.Afunctuion f(z) is rational iff it is meromorphic in the extended complex plane c_∞.

5.The range of a non constant entire function is a dense subset of C.

6. If g(z) takes values arbitrarily close to every complex number in every7 neighborhood of ∞.

7.If f is an entire function such that 0∈ $f(C)$and 1∈f(C) .Then f is constant .

8.Let f(z) be meromorphicinC and their exist a natural number n,m> 0 and R > 0 such that $|f(z)|$ ≤M$|z|^n$.Then f is rational function.

9.Solvei) f(z)=$\frac{-\log(1-z)}{z}$

ii) f(z)=$\frac{z}{e^z-1}$

SECTION C – 10 MARKS

1.State and prove Riemann's Removable singularity theorem.

2.State and prove Picard's little theorem.

3.State and prove Picard's Great theorem.

4. State and prove Weaker form of Picard's Great theorem.

5. State and prove Casorati Weiestress theorem.

6. State and prove Picard's little theorem for meromorphic function.

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TOPOLOGY – DMA 32

UNIT:1

2 Marks:

1. Define Topological space.
2. Define open set.
3. Define Basis topology.
4. Define Trivial topology.
5. Define Standard topology.
6. Define Lower limit topology.
7. Define K topology.
8. Define Sub basis.
9. Define Order topology.
10. Define Intervals.
11. Define Rays.
12. Define Order relation.
13. Define Comparability.
14. Define Non Reflexibility.
15. Define Transitivity.
16. Define Product topology.
17. Define projection.
18. Define open maps.
19. Define Subspace topology.
20. Define closed set.
21. Define limit point.
22. Let X={a,b,c,d} and B={{a},{b},{c}}.Then B is a basis for a topology.

23. To show that τ_y is a topology.

5 Marks:

1. Let X be a set and let $\boldsymbol{B}$ be a basis for a topology on X, then τ equals the collection of all union of elements of $\boldsymbol{B}$.
2. If $\boldsymbol{B}$ isthe basis for a topology on X and $\boldsymbol{\mathcal{C}}$ isthe basis for a topology on Y, then the collection $\boldsymbol{\mathcal{D}}$ = { B x C / B ε $\boldsymbol{B}$, C ε$\boldsymbol{\mathcal{C}}$ } isa basis for a topology on X x Y.
3. Let X be a topological space. Suppose that $\boldsymbol{\mathcal{C}}$is a collection of open sets of X such that for each open sets U of X and each x in U, there is an element C of $\boldsymbol{\mathcal{C}}$such that xεC⊂U. Then $\boldsymbol{\mathcal{C}}$is a basis for the topology of X& the topology generated by $\boldsymbol{\mathcal{C}}$ is same as τ.
4. If A is a subspace of X and B is a subspace of Y. Then the product topology of A x B is the same as the topology A x B inherits as a subspace of X x Y.
5. Let A be a subset of the topological space X, let A' be the set of all limit points of A. Then $\overline{A}$ = A∪ A′.
6. Let Y be a subspace of X. Then a set A is closed on Y iff its equals the intersection of a closed set of X with Y.
7. Prove that every finite point set in a Hausdroff space X is closed.
8. Let X be a space satisfying the T_1 axiom, let A be a subset of X. Then the point x is a limit point of Aiff every nbd of x contains infinitely many points of A.
9. If X is a Hausdroff space, then a sequence of points of X converges to atmost one points of X.
10. Let the set $\{X_\propto\}$ be an indexed family of spaces, let $A_\propto \subset X_\propto$ for each ∝. If $\prod X_\propto$ is given either the product of the box topology then $\prod \overline{A}_\propto = \overline{\prod A_\propto}$.
11. Let f : A → $\prod_{\propto \in J} X_\propto$ be given by the equation f(a) = ($f_\propto$(a)) where $f_\propto$:A→$X_\propto$ for each ∝. Let $\prod X_\propto$ have the product topology then the fn. is continuous iff each fn. $f_\propto$ is continuous.
12. Let X be a topological space then the following condition holds.
    1. φ and X are closed.
    2. Arbitrary intersection of closed sets are closed.
    3. Finite union of closed sets are closed.

10 Marks:

1. Every simply ordered set is a Hausdroff space. The product of two Hausdroff space is Hausdroff space and the subspace of Hausdroff space is Hausdroff space.
2. The topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology on R, but are not comparable with one another.
3. Let $\boldsymbol{B}$ and $\boldsymbol{B}'$be basis for a topology τ andτ**′.** Then the following are equivalent.
   i) τ′is finer than τ **.**
   ii) For each x ε X and each basis element B ε $\boldsymbol{B}$ containing x, there is a element B' ε $\boldsymbol{B}'$such that xε $B' \subset \boldsymbol{B}$ **.**
4. Let X be an ordered set in the order topology. Let Y be an subset of X that is convex in X. The order topology on Y is the same as the topology Y as a subspace of Y.
5. Let A be a subset of the topological space X. Then
   a) Then x ε$\overline{A}$ iff every open set U containing x intersects A.
   b) Supposing the topology of X is given by a basis, then x ε$\overline{A}$ iff every basis element B containing x intersects A.

UNIT:2

2 Marks:

1.Define Continuity of a function.

2. Define homeomorphism.

3.State pasting lemma.

4.State maps into products.

5. Define product topology.

6. Define Box topology.

7. StateProduct space.

8. DefineMetric topology.

9. Define metric space.

10. Define uniform topology.

11. State Sequence lemma.

12. State Uniform limit theorem.

5 Marks:

13. State & Prove Pasting lemma.
14. Let X be a metric space with metric d. Define $\overline{d}$: X x X → R by the equation $\overline{d}$(x,y) = min { d(x ,y) , 1} then $\overline{d}$ is a metric that induces the same topology as d. The metric $\overline{d}$ is standard bounded metric corresponding to d.
15. Let X, Y be a topological space.
    a) If f: X→Y and g:Y→Z are continuous then the map g ∘ f :X→Z is continuous.
    b) The map f: X→Y is continuous, if X can be written as the union of open sets $U_\propto$ such that f /$U_\propto$ is continuous for each ∝ .
16. Let f: A→ XxY be given by the equation f(x) = (f_1(a) , f_2(a)). Then f is continuous iff the fn. f_1:A→X and f_2:A→Y are continuous.
17. The uniform topology $\mathbb{R}^J$ is finer than the box topology; these three topologies are all different if J is infinite.
18. State & Prove sequence lemma.
19. Let f : X → Y ; let X and Y be metrizable with metricesd_x and d_y. Then continuity of f is equivalent to the requirement that, given x ε X and given ε > 0 there exists δ >0 such that d_x(x,y) < δ implies d_y(f(x),f(y)) < ε.
20. Let X be a non empty compact hausdroff space, if X has no isolated point then X is uncountable.

10 Marks:

1.Let X and Y be a topological space. Let f : X → Y then the following are equivalent.

i) f is continuous.

ii) For every subset A of X one have f ($\overline{A}$) ⊂ $\overline{f(A)}$.

iii) For every closed set B of Y, the set $f^{-'}$(V) is closed in A.

iv) For each x ε X and each neighbourhood V of f(x), there is a neighbourhood U of x such that f (U) ⊂ V.

2.The topologies on $\mathbb{R}^n$ induced by the Euclidian metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

3.Let d and d'be two metrices on the set X; let τ andτ'be the topologies they induce, respectively. Then τ' is finer than τ iff for each x ε X and each $\in > 0$, there exist a $\delta > 0$ such that $B_{d'}(x, \delta) \subset B_d(x, \in)$.

4.Let f : X → Y. If the fn. F is continuous, then prove that for every convergent sequence $x_n \rightarrow x$ in X, the sequence $f(x_n)$ converges to f(x). Prove further that the converse hold if X is metrizable.

5.Let$\bar{d}(a,b)$ = min {| a-b |, 1} be the standard bounded metric on R. If x, y are two points of R^ω, define D(x,y) = sup $\{ \frac{\bar{d}(xi, yi)}{i} \}$. Then D is the metric that induces the product topology on R^ω.

6.State& Prove Uniform limit Theorem.

UNIT:3

2 Marks:

1.Define Connectedness.

2.what is locally disconnected.

3. Define linear continuous.

4. State Intermediate value theorem.

5. Define path connected.

6.Show that a path connected space x is connected.

7. Define components.

8. Show that the relation is an equivalence relation .

9. Define path components.

10.what is locally connected.

5 Marks:

1. Prove that connectedness is the topological property.
2. If Y is a subspace of X, a separation of Y is a pair of disjoint non empty sets A & B whose union is Y, neither of which contains a limit point of the other. The space Y is connected if there exist no separation of Y.
3. If the sets C & D form a separation of X, and if Y is a connected subspace of X , then Y lies entirely within either C or D.

4. Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$, then B is connected.
5. The components of X is connected disjoint, whose union is X such that each non empty connected subspaces of X intersects only one of them.
6. A space X is locally connected iff for every open set U of X, each component of U is open in X.
7. A space X is locally path connected iff for every open set U of X, each path component of U is open in X.

10 Marks:

1.Show that the union of a collection of connected subspaces of X that have a point in common is connected.

2.S.T the image of a connected space under a continuous map is connected.

3.S.T the finite Cartesian product of connected spaces is connected.

4.If L is a linear continuum in the order topology, then L is connected, and so are intervals and rays in L.

5.Let X be a topological space.

a) Let A be a subset of X. If there is a sequence of points of A converging to x then $x \in \overline{A}$. The converse holds if X is first countable.

b) Let $f : X \to Y$. If the fn. F is continuous, then prove that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to f(x). Prove further that the converse hold if X is first countable.

6. Assume that X has a countable basis then

a) Every open covering of X contains a countable sub collection covering X.

b) There exist a countable subset of X that is dense in X.

UNIT:4

2 Marks

1.Define Compact space.

2.Prove that the interval [0,1] is not compact.

3.State the tube lemma.

4. Define finite intersection property.

5.Prove that the tube lemma is certainly not true. If y is compact.

6. Prove that every closed interval in R is compact.

7. State lebesgue number lemma.

8. Define Isolated point.

9. Define uniform continuous.

10. Define Limit point continuous.

11. Define Locally continuous.

12. Define Compactness.

5 Marks:

1.If X is a topological space, each path component of X lies in a component of X. If X is locally path connected, then the components of X are the same.

2.A subspace of a first countable space is first countable and a countable product of first countable space is first countable. A subspace of a second countable space is second countable and a countable product of second countable space is second countable.

3.P.T the subspace of a Lindelof space need not be Lindelof.

4.P.T the product of a Lindelof space need not be Lindelof.

5.P.T the space $\mathbb{R}_l$satisfies all the countability axioms.

6.Let X be a topological space. Let one point sets in X be closed.

a) X is regular iff given a point x of X and angd U of x, there is a ngd V of x such that$\overline{V} \subset$ U.
b) X is normal iff given a closed set A and an open set U containing A, there is an open set V containing A such that $\overline{V} \subset$ U.

7.P.T every regular space with a countable basis is normal.

8.P.T every metrizable space is normal.

9.P.T every compact Hausdroff space is normal.

10.A subspace of a completely regular space is completely regular and a product of completely regular space is completely regular.

11.The subspace $\mathbb{R}_k$ is Hausdroff but not regular.

10 Marks:

1.A subspace of a Hausdroff space is Hausdroff space and a product of Hausdroff space is Hausdroff space. A subspace of a regular space is regular and a product of regular space is regular.

2.P.T every well ordered set X is normal in the order topology.

3.State& Prove Urysohn lemma.

4.State& Prove UrysohnMetrization theorem.

5.State& Prove Tietz extension theorem.

6.State& Prove Tube lemma.

UNIT:5

2 Marks:

1. Define Countability axiom.
2. Define Lindle of space.
3. Define Uryshon lemma
4. State imbedding theorem.
5. State Tietzextention theorem.
6. Define dense in X.
7. Write the three separation axioms.
8. Define separable.
9. State Uryshonmetriziation theorem.
10. Define Hausdroff.

5 Marks:

1. P.T every closed subspace of a compact space is compact.
2. P.T every compact subspace of a Hausdroff space is closed.
3. If Y is compact subspace of the Hausdroff space X & x_o is not in Y, then there exist disjoint open sets U & V containing x_o and Y.
4. S.T the image of a compact space under a continuous map is compact.
5. Let f: X→Y be a bijective continuous fn. If X is compact and Y is Hausdroff then f is the homeomorphism.

6. Prove that the product of two compact spaces is compact.
7. A subspace A of R^n is compact iff it is closed and is bounded in the Euclidean metric d or the square metric ρ.
8. State & Prove Uniform continuity theorem.
9. Let X be a non empty compact Hausdroff space. If X has no isolated points then X is uncountable.
10. Prove that compactness implies limit point compactness but not conversely.
11. S.T R is not compact.
12. The following subspace of R is compact X = {0} $\cup\{\frac{1}{n}$ / n∈ Z_+ }.

10 Marks

1.Let X be a topological space. Then X is compact iff for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property. The intersection of all the elements of $\mathcal{C}$ is not empty.
2.Let X be a simply ordered set having least upper bound property in the order topology, every closed interval in X is compact.
3.Let X be a topological space. Then prove that X is compact iff for every collection $\mathcal{C}$of closed sets in X having the finite intersection property, the intersection $\bigcap_{c\varepsilon\mathcal{C}} C$ of all elements of $\mathcal{C}$ is nonempty.

4.Prove that every closed interval in R is compact.
5.State& Prove Extreme value theorem.
6.State& Prove Lebesgue no. lemma.
7.Let X be a metrizable space, then the following are equivalent.
i) X is compact.
ii) X is limit point compact.
iii) X is sequentially compact.

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DIFFERENTIAL GEOMETRY – DMA 33

2 MARKS

UNIT I

1. Define local differential geometry
2. Define global differential geometry
3. Define Space curve.
4. Define c^m function.
5. Define c^∞ function.
6. Define c^w function.
7. Define regular parameter
8. Define path of the class r

9. Define curve
10. Obtain the equation of the circular helix $\vec{r} = (acosu, asinu, bu), -\infty < u < \infty$ replaced to S has parameter. Show that the length as one complete turn of the helix is $2\pi c$ where $c = \sqrt{a^2 + b^2}$
11. Define osculating Plane
12. Prove that they are linearly of $\vec{r}''(0) \neq 0$
13. Find the equation of the osculating plane at the point on the cubic equation by $\vec{r} = (, u, u^2, u^3)$
14. Find the osculating plane at the point O on the helix $x = acos\theta, y = asin\theta, z = c\theta$
15. Define Normal plane
16. Define principal normal
17. Define curvature
18. To find the magnitude and the direction or curvature K
19. Define Osculating circle
20. Define Binomial vector
21. Define Torsion
22. Prove that $\hat{b}' = -\tau\hat{n}$
23. State SerretFrenet formula
24. Define Involute
25. Define Evolute
26. Define Tangent surface
27. Define cylindrical Helix

UNIT II

1. Define Surface
2. Define explicit form of the surface
3. Monge's form of the surface
4. Define ordinary point
5. Define singularity point
6. Define essential singularity
7. Define Artificial singularity
8. Define surface of revolution
9. Define tangent plane Define normal plane
10. Define Anchor Ring

11. Define Helicoid
12. Prove that The metric is positive definite
13. Define direction co-efficient
14. Define Direction ratio
15. Define family of curve
16. Define orthogonal Trajectories
17. Define Isometric Correspondence

UNIT III

1. Define Geodesic
2. State Differential equation of the Geodesic
3. Prove that every helix on a cylinder is a Geodesic
4. Define Geodesic Parallel
5. Define Geodesic curvature
6. Prove that the Geodesic curvature vector of any curve as orthogonal to the curve
7. State Liouville's formula for formula for Geodesic curvature
8. State Gauss Bonnet theorem
9. Define Gaussian curvature
10. To find the total curvature of the sphere of radius a
11. State Minding theorem
12. Define Cristoffell's symbol of first kind
13. Define Cristoffell's symbol of second kind

UNIT IV

1. Define second fundamental form
2. Define Elliptic point
3. Define Parabolic point
4. Define Hyperbolic point
5. Find the normal curvature of the right helicoid $\vec{r} = (acosu, asinu, cv)$ at the point on it
6. State Meusnier theorem
7. Define Principal curvature

8. Define Lines of curvature
9. State Euler's theorem
10. Define DupineIndicatrix
11. Define Umbilic point
12. Define Asymptotic

UNIT V

1. Define Weingarten Equation
2. Define Mainardi-Codazzi Equation
3. State Fundamental Existence theorem for surface
4. State Hilbert's lemma

5 MARKS

UNIT I

1. Define the following (i) Function of class m (ii) c^{∞} - function (iii) Analytic function.
2. Define the following (i) Class of a vector valued function (ii) path (iii) Equivalent path.
3. Write the equivalent parametric representation for the circular helix
   $\vec{r}$= (a cosu, a sinu, cu) 0$\leq u < \pi$.
4. Find the equation of the osculating plane.
5. S.T if a curve is given interms of a general parameter u, then equation of the osculating plane is $[\vec{R}-\vec{r},\dot{\vec{r}},\ddot{\vec{r}}]$=0.
6. Find the equation of the osculating plane at the point of inflexion.
7. State and proveSerret-Ferenet formula.
8. Prove that necessary and sufficient condition that a curve to be a straight line is K=0 at all points.
9. Prove that necessary and sufficient condition that a curve is that $\tau = 0$ at all points.
10. P.T $[\vec{r}', \vec{r}'', \vec{r}''']$=k^2 τ.

UNIT II

1. Define the following (i) Explicit form of surface (ii) vector form of surface (iii) Monge's form of surface.

2. P.T the parametric equation of a surface are not unique.
3. Define the following (i) singularity (ii) Essential singularity (iii) Artificial singularity.
4. P.T an ordinary point is unaltered by a proper parametric transformation.
5. S. T a proper parametric transformation either leaves every normal unchanged or reverses every normal.
6. Find the surface area of the anchor ring.
7. Obtain the equation of the general surface of revolution.
8. Find the angle between the two directions on a surface at a point P having directions (l,m) & (l',m')
9. Find the coefficients of the direction which make an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l,m).
10. P.T the metric is invariant under parametric transformation.

UNIT III

1. Necessary and sufficient conditions for the curve to be geodesic $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$.
2. PT the curves of family $\frac{v^3}{u^2}$= constant are geodesic on a surface with metric $V^2du^2 - 2uvdudv + 2u^2dv^2$ [u>0, v>0].
3 PT on the general surface a necessary and sufficient condition that the curve v=c be a geodesic is $EE_2+FE_1-2EF_1=0$.
4 PT on the general surface a necessary and sufficient condition that the curve u=c be a geodesic is $GG_1+FG_2-2GF_2=0$.
5 Derive the normal property of geodesic.
6 A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is a geodesic.
7 Prove that every helix on a cylinder is a geodesic.
8 Prove that the geodesic curvature vector of any curve is orthogonal to the curve.
9 Define (i) Geodesic triangle. (ii) Geodesic curvature vector.
10 PT components λ, μ of the geodesic curvature vector are given by $\lambda = \frac{1}{H^2}U/V'\frac{\partial T}{\partial v'}$,

$$\mu = \frac{1}{H^2}V/U'\frac{\partial T}{\partial u'},$$

11. To find the Gaussian curvature of the sphere of radius a.
12. Find the local curvature of the sphere of radius a.
13. By using Gauss-Bonnet theorem find the Gaussian curvature of the sphere of radius a.
14. Find the Gaussian curvature at the point (u,v) of the anchor ring and also Verify that the total curvature of the whole surface is zero
15. State and prove the Meusnier's theorem.
16. Show that the anchor ring contains all three types of points.
17. Derive the Principle curvature.
18. Derive the Mean curvature.
19. PT the Principle directions are orthogonal.

UNIT IV

1. Prove that the tangents to the edge of regression are characteristic lines.
2. Prove that osculating plane at any point on the edge of regression is identical with corresponding plane a one parameter family of planes of a developable.
3. Prove that the edge of regression of the osculating developable is the curve itself.
4. Prove that the edge of regression of polar developable is the locus of centers of spherical curvature of the given curve.
5. Prove that the asymptotic lines are orthogonal when the surface is a minimal surface.
6. Show that the lines of curvature form an isothermal net on a minimal surface.
7. Find the equation of the ruled surface.
8. Find the fundamental coefficients of the ruled surface.
9. Find the Gaussian curvature of the ruled surface.
10. Find the equation of the asymptotic lines on a ruled surface.

UNIT V

1. Derive Weingarten Equation
2. Derive Weingarten Equation in tensor form
3. Derive Mainardi – Codazzi Equation
4. Show that the equation of Gauss
5. State and prove Hilbert's lemma

6. Prove that compact surface whose points are umbilic

10MARKS

UNIT I

1. State and prove Fundamental Existence theorem for space curves.
2. State and prove Uniqueness theorem for space curves.
3. Obtain the equation of Involute and Evolute.
4. Show that Involutes of a circular helix are plane curve.
5. Find the curvature and torsion of an involute of a curve C.
6. P.T the product of the torsions of C and C_1 at corresponding points is equal to the product of the curvature at these points.
7. Show that the radius of curvature of the Locus of the centre of curvature of a curve is given by$[\{\frac{\rho^2\sigma}{R^3}\frac{d}{ds}\left(\frac{\sigma\rho'}{\rho}\right)-\frac{1}{R}\}^2+\frac{\rho'\sigma^4}{\rho^2R^4}]^{\frac{-1}{2}}$.
8. Find the equation of osculating circle and osculating sphere.
9. Obtain curvature and torsion of the curve of intersection of two quadric surface $ax^2+by^2+cz^2=1$; $a'x^2+b'y^2+c'z^2=1$.
10. Show that the lenth of the common perpendicular d of the tangent at 2 near points distance S a path is approximately given by $d=\frac{k\tau s^3}{12}$

UNIT II

1. Find a surface of revolution which is isometric with a region of the right helicoid.
2. A surface of revolution is defined by the equation x = cosucosv,y=cosusinv , z=-sinu+logtan $(\frac{\pi}{4}+u/2)\,wh$ ere $0<u<\frac{\pi}{2}$ 0< v<2 π .show that the metric is $\tan^2u du^2+\cos^2u dv^2$.also prove that the region $0<u<\frac{\pi}{2}$ 0< v<2 π is mapped isometrically on the region$\frac{\pi}{3}<u'<\frac{\pi}{2}$, 0< v'<2 π by the correspondence $u'=\cos^{-1}(1/2\cos u)$ $v'=2v$.
3. Show that the curves $du^2-(u^2+a^2)dv^2=0$ on the right helicoids form on orthogonal net.

4. A helicoids is generated by the screw motion of the straight line which meets the axis at an angle $\propto$
   (i) Find the orthogonal trajectories of the generators.
   (ii) Find also the metric of the surface refered to the generators and their orthogonal trajectories as parametric curves.

UNIT III

1. State and prove Gauss-Bonnet theorem.
2. State and prove Liouville's formula for k_g.
3. Derive the geodesic differential equations.
4. Find the geodesic on a surface of revolution.
5. Derive the differential equation of the geodesics using normal property.
6. Prove that the geodesic curvature of a geodesic is zero.
7. Prove that if the orthogonal trajectories of the curve v= constant are geodesics then H^2/E is independent of u.
8. State and Prove Minding theorem.
9. State and prove Rodrigue's formula.
10. State and prove Euler's theorem.
11. Prove that the Necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature is zero.
12. Find the equation of the dupinindicatrix.

UNIT IV

1. Prove that the cross ratio of the four points in which a generator is met by four given curved asymptotic lines is the same for all generators.
2. Prove the properties of the Parameter of distribution of the ruled surface.
3. Find the formula which determines the position of the central point on each generators.
4. Find the formula for K interms of p & v.
5. Prove that if there is a surface of minimum area passing through a closed space curve then it is a minimal surface.

6. Prove that let C be a curve lying on a surface and let P be any point on C then the characteristic line at P of the tangential developable of C is in the direction conjucate to that of the tangent to Cat P.
7. State and prove Monge'stheorem.
8. Prove that the necessary and sufficient condition for a surface to be developable is that it's Gaussian curvature.

UNIT V

1. Derive Gauss equation
2. State and prove fundamental existence theorem for surfaces
3. Derive Mainardi – Codazzi Equation

****

Latex – DEMA 34A

UNIT – 1

2 - mark

1. Explain previewer.

2. what is programming language?

3. what is called backslash?

4. Explain space.

5. Explain Dashes.

6. ExplainQuotes.

7. Write few Accents in Latex.

8.Write few Special symbols in Latex.

9. Explain font.

10. Writetype size in Latex.

11. WritePaper size.

12. Explain Font size.

13. What are the Paper sizes in latex.

14. How to construct Title in Latex .

15. How to construct Abstract in Latex .

16. Write Options for natbib.

17. Write short notes on Bibliographic database

18. Write short notes on Partial citations.

19. ExplainMultiple citations

20.Write short notes on parenthetical citations

5 -mark

1. Explain simple typesetting.

2. Write brief notes on Type style.

3. Explain Special symbols in Latex.

4. Write brief notes onPage formats.

5. Explain Multiple citations.

6. Explain Numerical mode

7. Explain Partial citations

8. Explain Citations aliasing

9. Write notes on NATBIB

10. Explain Suppressed parentheses.

11. Write Selecting citation style and punctuation.

12. Explain dividing the document.

10 – mark

1. Explain Basic commands in Latex.

2.ExplainBibliography.

3.Write notes on dividing the document.

4.Explain parts of a document.

5.Write notes on formatting lengths.

6. Explain page numbering.

7. Write full notes ondocument.

8.Explain simple typesetting.

9. Explain Special symbols.

10. Write notes on Text positioning.

UNIT – 2

2 – mark

1. Define Additional entries
2. ExplainTypesetting a contents list
3. DefineMultiple tables of contents
4. Explain index
5. Define Simple index entries
6. Explain Sub entries
7. Explain Page ranges
8. What is cross-references ?
9. Explain Controlling in Latex.
10. Define Controllingthe presentation form.

5 – mark

1. Write notes on Printing special characters.
2. Write notes on Saying it with bullets.
3. Explain features of Creating floating figures.
4. How to form Matrices in Latex.
5. write the Latex command for below equation

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

6. Explain LR boxes in Latex.
7. Write Latex command for following sentence.

Let’s review the notation

• (0,1) is an open interval

• [0,1] is a closed interval

8. Write Latex command for following sentence.

Let us take stock of what we have learnt

TEX A typesetting program.

Emacs A text editor and also a programming environment and a lot else besides.

AbiWord A word processor.

9. Write Latex command for following sentence.

Addition of numbers satisfies the following conditions:

(A1) It is commutative

(A2) It is associative

(A3) There is an additive identity

(A4) Each number has an additive inverse

10. Write the tex for following Latex command

Some mathematicians elevate the spirit of Mathematics to a kind of intellectual aesthetics. It is best voiced by Bertrand Russell in the following lines.

\begin{quote}

The true spirit of ....from which all great work springs.

\end{quote}

11. Explain glossary.

10 – mark

1. Explain borrowed words.

2. Write notes on displayed text.

3. Write notes on poetry in typesetting.

4. Explain making lists in Latex.

5. Explain descriptions.

6. Write notes on definitions.

7. Explain when order matters in Latex .

8. How to making lists in Latex ?

9. Write Latex command for following sentence.

One should keep the following in mind when using TEX

• TEX is a typesetting language and not a word processor

• TEX is a program and and not an application

• Theres is no meaning in comparing TEX to a word processor, since the design purposes are different Being a program, TEX offers a high degree of flexibility.

10 . Write notes on definitions.

11. Write Latex command for following sentence.

• The first item in the first level

• the second item in the first level

– The first item in the second level

– the second item in the second level

∗The first item in the third level

∗the second item in the third level

· The first item in the fourth level

·the second item in the fourth level.

UNIT – 3

2 – mark

1. Define rows.

2.Define columns

3. Explainkeeping tabs in Latex.

4. Define Pushing

5. Define popping.

6.How to construct tables?

7.How to construct Enhancements to the tabular?

8.Explain array package

9. Definemultirow package.

10.Explain Multipage tables.

11. Write notes on package longtable.

12. Define tabular.

5 – mark

1. Write difference between tabbing and tabular.

2.Write difference between Superscripts and subscripts

3. Explain command for Roots in Latex.

4.ExplainMathematical symbols in latex .

5.Write Latex command for following sentence.

Program : TEX Author : Donald Knuth Manuals :

Title Author Publisher

The TEXBookDonald Knuth Addison-Wesley

The Advanced TEX Book David Salomon Springer-Verlag

6. Write Latex command for following sentence.

The table below shows the sizes of the planets of our solar system.

Planet	Diameter(km)
Mercury	4878
Venus	12104

Earth	12756
Mars	6794
Jupiter	142984
Saturn	120536
Uranus	51118
Neptune	49532
Pluto	2274

As can be seen, Pluto is the smallest and Jupiter the largest.

7. Write Latex command for following sentence.

Planet Features Mercury Lunar like crust, crustal faulting, small magnetic fields.

Venus Shrouded in clouds, undulating surface with highlands, plains, lowlands

and craters. Earth Ocens of water filling lowlands between continents,

unique in supporting life, magnetic field.

Mars Cratered uplands, lowland plains, volcanic regions.

Jupiter Covered by clouds, dark ring of dust, magnetic field.

Saturn Several cloud layers, magnetic field, thousands of rings.

Uranus Layers of cloud and mist, magentic field, some rings.

Neptune Unable to detect from earth.

Pluto Unable to detect from earth.

8.Write Latex command for following sentence.

The sequence (x_n) defined by

$x_1 = 1$, $x_2 = 1$, $x_n = x_n{-}1 + x_n{-}2$ $(n > 2)$

is called the Fibonacci sequence.

9.Write Latex command for following sentence.

For real numbers x and y, define an operation □ by

$X \square\ Y = x_2 + y_2$

10.Write Latex command for following sentence.

$(a+b+c+d+e+ f)^2= a^2 +b^2 +c^2 +d^2 +e^2 + f^2$

$+2ab+2ac+2ad+2ae+2af$

$+2bc+2bd+2be+2bf$

$+2cd+2ce+2cf$

$+2de+2df$

$+2ef$

11.Write Latex command for following sentence.

Thus x, y and z satisfy the equations

$x+ y-z = 1$

$x-y+z = 1$

and by hypothesis

$x+ y+z = 1.$

12.Write Latex command for following sentence.

Compare the sets of equations

$\cos^2 x+\sin^2 x = 1$ $\cosh^2 x-\sinh^2 x = 1$

$\cos^2 x-\sin^2 x = \cos 2x$ $\cosh^2 x+\sinh^2 x = \cosh 2x.$

10 – mark

1. Explain custom commands.
2. Write notes on Single equations.
3. Write notes on Groups of equations.
4. Write notes on Numbered equations.
5. How to make Matrices.
6. Explain Dots in Latex

7. ExplainDelimiters in Latex

8. Explain Affixing symbols—over or under.

9. Explain the many faces of mathematics.

10. Explain designer theorems—the amsthm package.

11. . ExplainCustom made theorems.

UNIT – 4

2 – mark

1. write uses of marginal notes

2. Explain rows and columns in Latex.

3. What are the parameters Footnote style

4. Write the difference between tabbing and tabular.

5. Define Cross reference.

6. Specify Multipage tables in Latex.

7. What is the command for put square root in Latex.

8. What is the command for mathematical equation in Latex.

9. How to Constructing tables in Latex.

10. What are the contributions of h,t,b,p in figure placement?

11.Write the example of using Dots in Latex.

12. Explain LR boxes in Latex.

5 – mark

1. Explain features of Creating floating figures.

2. How to form Matrices in Latex.

3. write the Latex command for below equation

$(a+b)^2 = (a+b)(a+b)$

$= a^2 + ab + ba + b^2$

$= a^{2} + 2ab + b^{2}$

4. Explain LR boxes in Latex.

5. Write the example of using Dots in Latex

6.write the Latex command for below equation

$(x+y)^{2} - (x-y)^{2} = ((x+y)+(x-y))((x+y)-(x-y)) = 4xy$

7. write the equation for the below latex command

```
\begin{equation*}

\frac{4}{\pi} = 1+\cfrac{1ˆ2}{2+

\cfrac{3ˆ2}{2+

\cfrac{5ˆ2}{2+\dotsb}}}

\end{equation*}
```

8. Write the some points for the recognized typesetting standards in mathematics.

9. write the Latex command for below equation

$(a+b+c+d+e+f)^{2} = a^{2} + b^{2} + c^{2} + d^{2} + e^{2} + f^{2}$

$+2ab+2ac+2ad+2ae+2af$

$+2bc+2bd+2be+2bf$

$+2cd+2ce+2cf$

$+2de+2df$

$+2ef$

10. Explain Groups of equations.

11. Write the tex for the following latex command

```
\begin{align}

x+y-z& = 1\\

x-y+z& = 1

 \end{align}
```

12. Write the tex for the following latex command

The system of equations

```
\begin{align*}
x+y-z& = 1\\
x-y+z& = 1\\
x+y+z& = 1
\end{align*}
can be written in matrix terms as
\begin{equation*}
\begin{pmatrix}
1 & 1 & -1\\
1 & -1 & 1\\
 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
x\\
y\\
z
\end{pmatrix}
=
\begin{pmatrix}
1\\
1\\
1
\end{pmatrix}.
```

\end{equation*}

Here, the matrix

$\begin{pmatrix}

1 & 1 & -1\\

1 & -1 & 1\\

1 & 1 & 1

\end{pmatrix}$

is invertible.

10 – mark

1. How to figure placement in Latex.
2. Explain nested boxes.
3. Explain paragraph boxes with specific height.
4. write notes on several kinds of boxes.
5. write notes on several kinds of boxes.
6. Explain designer theorems—the Amsthm package.
7. Define the many faces of mathematics.
8. Explain Custom made theorems.
9. write notes on typesetting theorems.
10. Explain new mathematics operators in Latex.
11. Write the tex for the following latex command

\begin{tabular{l|c|r|}

\hline

\multicolumn{3{|c|}{Sample Tabular}

\hline

col head & col head & col head

\hline

Left &centered & right \\

\cline{1-2}

aligned& items & aligned \\

\cline{2-3}

items& items & items \\

\cline{1-2}

Left items & centered & right aligned

\hline

\end{tabular}

UNIT – 5

2 – mark

1. what is cross references?

2. How to use Cross references in math?

3. Explain pointing to a page

4. Write the uses of the package varioref.

5. How topointing outside in Latex?

6. Explain the package XR?

7. What to dolost the keys?

8. Write the uses lablst.tex.

9. Define footnotes.

10. Define marginpars

11. Define endnotes.

5 – mark

1. Explain Footnotes in tabular material.

2. Explain Customizing footnotes.

3. Write the Latex command for the following tex
   In the classical syllogism
   (1) All men are mortal.
   (2) Socrates is a man.
   (3) So Socrates is a mortal.
   Statements (1) and (2) are the premises and statement (3) is the conclusion.
4. Write the tex for the following latex command
   ```
   \begin{table}[h]
   \begin{center} \setlength{\extrarowheight}{5pt}
   \begin{tabular}{|c|c|c|c|}
   \hline Value of $x$ & 1 & 2 & 3\\
   \hline Value of $y$ & 1 & 8 & 27\\
   \hline
    \end{tabular}
   \caption{Observed values of $x$ and $y$}\label{tabxy}
   \end{center}
    \end{table}
    Two possible relations betweeen $x$ and $y$ satisfying the data in Table\ref{tabxy} are
   $y=x^3$ and $y=6x^2-11x+6$
   ```
5. Write the tex for the following latex command
   ```
   \begin{align}
   (x+y)^2&=x^2+2xy+y^2\label{sum}\\
    (x-y)^2&=x^2-2xy+y^2\tag{\ref{sum}a}
    \end{align}
   ```
6. Write the Latex command for the following tex.

Table XIII.1: PostScript type 1 fonts

Courier[a]cour, courb, courbi, couri

Nimbus[b]unmr, unmrs

10 – mark

1. Explain End notes in Latex .

2.. write the Latex command for below text.

Definition IX.2.1. A triangle is the figure formed by joining each pair of three non collinear points by

line segments.

Note IX.2.1. A triangle has three angles. 1note

Theorem IX.2.1. The sum of the angles of a triangle is 180◦.

Lemma IX.2.2. The sum of any two sides of a triangle is greater than or equal to the third.

3.. How to Constructing tables in Latex.

4..How to Use graphics in L ATEX.

5. Explain Footnotes in tabular material.

6. Write the notes on Footnote style parameters.

7. Write the uses of marginal notes.

8. Write the Style parameters for marginal notes.

9. Explain designer theorems—the Amsthm package.

10. Define the many faces of mathematics.

11. write the Latex command for below text.

(XII.1) $(x+ y)^2 = x^2 +2xy+ y^2$

Changing y to−y in Equation (XII.1) gives the following

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6. Define Data members.

Programming Using C++ - DOCS 35B

PART – A (2 MARKS)

1. Define OOPs.
2. Define Objects.
3. What are the features of Object oriented programming.
4. Define Encapsulation and Data hiding.
5. Define Data Abstraction.
6. Define Data members.

7. Define Member functions.
8. Define Inheritance.
9. Define Polymorphism.
10. List and define the two types of Polymorphism.
11. Define Dynamic Binding.
12. Define Message Passing.
13. List some benefits of OOPS.
14. List out the applications of OOP.
15. What is the return type of main ()?
16. List out the four basic sections in a typical C++ program.
17. Define Token. What are the tokens used in C++?
18. Define identifier. What are the rules to be followed for identifiers?
19. State the use of void in C++.
20. Define an Enumeration data type.
21. Define reference variable. Give its syntax.
22. List out the new operators introduced in c++.
23. What is the use of Scope resolution operator?
24. List out the memory referencing operators.
25. Define Implicit Conversion.
26. What is call by reference?
27. What are inline functions?
28. State the advantages of Default Arguments.
29. Define Function overloading.
30. Define friend function.
31. Write the limitations/ disadvantages of C++
32. Define Constructor.
33. List some of the special characteristics of constructor.
34. Give the various types of constructors.
35. What are the ways in which a constructor can be called?

36. What is meant by dynamic initialization of objects.

37.Define Destructor.

38.List some of the rules for operator overloading.

39.What are the types of type conversions?

40.What are the conditions should a casting operator satisfy?

41.How the objects are initialized dynamically?

42.Define abstact class.

43.Define virtual base class

44.What are types of inheritance?

45.Give the syntax for inheritance.

46.Define single inheritance.

47.Define multi-level inheritance.

48.Define multiple inheritance.

49.What is an abstract class?

50.Define manipulators and also mention the manipulators that are used in C++.

51.What is the need for streams?

52.List some predefined streams.

53.What are the possible types that a file can be defined?

54.What are the two methods available for opening the files?

55.What is global namespace?

56.Write any four operations possible on string objects.

UNIT -1 TO 5 (5 marks)

1. Explain the different types of polymorphism.
2. Explain Multilevel and hybrid Inheritance.
3. Describe Pure Virtual function with an example.
4. Write a C++ program using this pointer.
5. Write a C++ program for calculating the are of rectangle and circle using run time polymorphism (5)

6. Explain the basic concepts of Object oriented programming
7. Explain the use of constant pointers and pointers to constant with an example.
8. State the differences between class and struct and also illustrate with an example.
9. What are the difference between pointers to constants and constant to pointers?
10. Write a C++ program using inline function. \
11. Write a C++ program to illustrate the static function
12. Explain about call by reference and return by reference with program.
13. Explain Nested classes and local classes with an example
14. Write a program to evaluate the following function

    Sin(x) = x - x3 /3! + x5 /5! – x7 /7! + ….

15. Explain the structure of C++ program
16. Explain in detail about formatted and unformatted console I/O operations.
17. . Write about declaring member function inside and outside a class
18. Explain the copy constructors with an example.

UNIT -1 TO 5 (10 marks)

1. Explain various types of Inheritance.
2. Write a C++ program using dynamic_const.
3. Write a program in C++ to read two strings and perform the following string manipulation function .
4. (A)Find the long string Compare the two strings
   Concatenate them b) Explain in detail about dynamic objects. How are they created

5. Explain the basic concepts of Object oriented programming
6. Explain briefly about function overloading with a suitable example.
7. Explain Nested classes and local classes with an example
8. Write a program to explain the concept of array of objects.
9. Explain explicit Constructors, Parametrized Constructors, and multiple Constructors
   a. with suitable example.
10. How to achieve operator overloading through friend Function?

11. Write a program to add two complex numbers using operator overloading concept
12. Write a C++ program to find the area of various 2D shapes such as square, rectangle, triangle, circle and ellipse using function overloading.
13. Explain about Formattted and Unformatted IO with suitable Example
14. What is manipulator? Difference between manipulators and ios Function?
15. Explain the process of open,read,write and close files?
16. Explain the role of seekg(),seekp(),tellg(),tellp(),function in the process of random access in a binary file .

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