KRISHNASAMY COLLEGE OF SCIENCE, ARTS & MANAGEMENT FOR WOMEN

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DEPARTMENT OF MATHEMATICS

QUESTION BANK 2021-2022

I- B.SC MATHEMATICS

SEMESTER I

ALGEBRA – CMA 11

UNIT I

1. Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1 and hence solve it.
2. Incerse the roots of the equation equation $x^4 + 12x^3 - 7x^2 + 22x + 24 = 0$
3. Remove the second term form the $x^3 - 6x^2 + 11x - 6 = 0$
4. Solve the equation $x^3 – 3x^2 – 6x + 8 = 0$ if the roots are in A.P.
5. If α, β and γ are roots of the equation $x^3 - 5x^2 - 2x + 24 = 0$ find the value of
   i) $\Sigma\ \alpha_2\beta$ ii) $\Sigma\ \alpha^2$ iii) $\Sigma\ \alpha^3$ iv) $\Sigma\ \alpha^2\beta^2$
6. Solve the equation $3x^3 – 26x^2 + 52x – 24 = 0$ if the roots are in G.P.
7. Remove the fractional coefficients from the equation $x^3 – 1\backslash 2\ x^2 + 3\backslash 2\ x – 1 = 0$
8. Find the equation whose roots are reciprocals of the roots of $x^4 – 5x^3 + 7x^2 + 3x – 7 = 0$
9. Find the equation whose roots are the roots of $x^4 – 5x^3 + 7x^2 – 17x + 11 = 0$ each diminished by 4.
10. Find the equation whose roots are those of $3x^3 – 2x^2 + x – 9 = 0$ each diminished by 5.
11. Remove the second term from equation $x^4 – 8x^3 + x2 – x + 3 = 0$
12. Remove the third term of equation $x^4 – 4x^3 – 18x^2 – 3x + 2 = 0$, hence obtain the transformed equation in case h =3.
13. Transform the equation $x^4 + 8x^3 + x – 5 = 0$ into one in which the second termis vanishing.
14. Solve Solve the equation $x^4+16x^3+83x^2+152x+84 = 0$ by removing the second term.
15. Solve the biquadratic $x^4 + 12x – 5 = 0$ by Descarte's method.
16. Solve $x^4 – 8x^2 – 24x + 7 = 0$ by Descarte's method.
17. Obtain the relation between the roots and coefficients of general polynomial equation $a^0x^n + a^1x^{n-1} + a^2x^{n-2} + \text{-----} + a^{n-1}\ x + a^n = 0$.
18. Solve the equation $x^3 – 5x^2 – 16x + 80 = 0$ if the sum of two of its roots being equal to zero.
19. Solve the equation $x^3 – 3x^2 + 4 = 0$ if the two of its roots are equal.
20. Solve the equation $x^3–5x^2 –2x+24 = 0$ if the product of two of the roots is 12.
21. Solve the equation x3 – 7x2 + 36 = 0 if one root is double of another.

22. Find the condition that the roots of the equation $x^3 –px^2+qx–r = 0$ are in A.P.

23. Find the condition that the cubic equation $x^3 + px^2 + qx + r = 0$ should have two roots α and β connected by the relation $\alpha\beta + 1 = 0$
24. If α, β and γ are roots of the cubic equation $x^3 + px^2 + qx + r = 0$ find the value of i) $\Sigma \alpha^2\beta$ ii) $\Sigma \alpha^2$ iii) $\Sigma \alpha^3$ iv) $\Sigma \alpha^2\beta 2$
25. If α, β and γ are roots of the cubic equation $x3 + px^2 + qx + r = 0$ find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$.
26. If α, β, γ and δ are roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of the following symmetric functions
    i) $\Sigma \alpha^2\beta$ ii) $\Sigma \alpha^2$ iii) $\Sigma \alpha^3$
27. If α, β, γ and δ are roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$,
find the value of the following symmetric functions i) $\Sigma \alpha^2\beta\gamma$ ii) $\Sigma \alpha^2\beta^2$ iii) $\Sigma \alpha^4$
28. the equation whose roots are the reciprocals of the roots of$x^4 – 3x^3 + 7x^2 + 5x – 2 = 0$.
29. If sum and product of roots of a quadratic equation are 1 and –1 respectively the required quadratic equation is?
30. Roots of equation $x^3 – 3x^2 + 4 = 0$ are 2, 2, -1, so the roots of equation $x^3 – 6x^2 + 32 = 0$ are?
31. Roots of equation $x2 + 2x + 1 = 0$ are -1, -1 so the roots of equation $x^3 + 6x + 9 = 0$ are?
32. Roots of equation $x2–2x+4=0$ are 2, 2 so the roots of equation $4x2–2x+1=0$ are?
33. Find the 6th power of the roots $x^3 - x^4 + 1 = 0$.
34. Prove that $x^4 - x^3 + 1 = 0$. has one negative root and 2 imaginary roots.
35. Solve $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ if 1+I is a root.
36. Diminish the equqtion $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1.
37. Find the value of $\sum \alpha^2$ if $x^3 - 6x^4 + 11x - 6 = 0$.
38. Prove that $x^7 - 4x^4 + 2x^3 - 1 = 0$ has atleast 4 imaginary roots.
39. Solve $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ whose roots are in A.P.
40. Remove second term of the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$.
41. If α, β, γ are the roots of the equation then find the equqtion whose roots are $\sum \alpha + \beta$.
42. Solve $x^4 + 2x^3 - 5x^2 - 6x + 2 = 0$ if $1+\sqrt{-1}$. is one of the root.
43. $x^3 + 2x + 3 = 0$ has one negative root and two imaginary root?.
44. Solve $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$.
45. Find the sixth power of $x^4 - x^3 - 7x^2 + x + 6 = 0$.
46. The equation $x^4+4x^3–2x^2–12x+9 = 0$ has two pairs of equal roots, find them.

47. Change the signs of the roots of the equation $x^7 + 5x^5 - x^3 + x^2 + 7x + 3 = 0$

48. Transform the equation $x^7 - 7x^6 - 3x^4 + 4x2 - 3x - 2 = 0$ into another whose roots shall be equal in magnitude but opposite in sign to those of this equation.

49. Change of the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another the

50. coefficient of whose highest term will be unity.

UNIT II

1. State and prove Descarts rule.
2. Show that $x^{10} + 10x^3 + x - 4 = 0$ has 8 imaginary roots.
3. Show that $x^6 + 3x^2 - 5x + 1 = 0$ has atleast 4 imaginary roots.
4. Show that $x^4 + 2x^2 + 3x - 9 = 0$ has 1 possitive root and 1 negative roots.
5. State Newtons formula.
6. Find the real root of the equation $x^3 + 6x - 2 = 0$.
7. Calculate the places of decimal the positive root of the equation $x^3 + 24x - 50 = 0$.
8. Evaluate $\sqrt{12}$ to Newtons metur decimal places by Newtons method.
9. Find by Newtons method negative root of the equation $x^3 - 21x + 35 = 0$ correct to three decimal places.
10. Find the roots of the equation $x^3 - 5x + 3 = 0$ the roots lies between 1 & 2.
11. Calculate the places of decimal the positive root of the equation $x^3 + 24x - 50 = 0$. Using Horners Method.

UNIT III

1. Define Symmetricic and Skew Symmetric matrices.
2. Verify that the matrix is symmetric or not $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.
3. Show that every square matrix can be uniquely expressed as the sum of the symmetric and skew symmetric matrices.
4. Express $\begin{pmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{pmatrix}$ as the sum of the symmetric and skew symmetric matrices.
5. Define Hermitian and Skew Hermitian matrices.
6. Check whether A = $\begin{pmatrix} 3 & 4-5i \\ 4-5i & 6 \end{pmatrix}$
7. Show that $\begin{pmatrix} 3 & 1+2i \\ 1-2i & 2 \end{pmatrix}$
8. Define Orthogonal matrix.
9. Stow that $1/3 \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ is Orthogonal.
10. Define Unitary marix.

11. Prove that A = $\begin{pmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{pmatrix}$ is Unitary.
12. State Cayley Hamilton Theorem.
13. Verify Cayley Hamilton Theorem for $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$.
14. Verify Cayley Hamilton Theorem for $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.
15. Find The Inverse of $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -1 \end{pmatrix}$ using Cayley Hamilton Theorem.
16. Define Eigen values and Eigen Vectors.
17. Find the w eigen values and eigen vector: $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$.
18. Find the w eigen values and eigen vector: $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{pmatrix}$.
19. If $\lambda_1, \lambda_2, \lambda_3$ are the eigen roots of A, find λ_3 of A = $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{pmatrix}$.
20. Define similar matrices.
21. Diagonalize the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.
22. Show that $1/\sqrt{7} \begin{pmatrix} 1+i & 2+i \\ 2-i & -1+i \end{pmatrix}$ is unitary.
23. Show that $\begin{pmatrix} \sin\theta & \cos\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
24. Verify Cayley-hamilton theorem for the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{pmatrix}$
25. Determine the Eigen values and eigen vectors of A = $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$.
26. Determine the Eigen values and eigen vectors of A = $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.
27. Diagonalize The matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

UNIT IV

1. $(x+a)^n$ =?
2. Expand
3. $(1+x)^n$
4. $(1-x)^n$

5. $(1+x)^{-n}$
6. $(1-x)^{-n}$
7. Find the Coefficient of x^n in the expansion of $(1/ 1-x^2)$
8. Expand
9. $(1+x)^{-1}$
10. $(1-x)^{-1}$
11. $(1+x)^{-2}$
12. Find the Coefficient of x^n in the expansion of $(2+3x)^{-3}$
13. Find the Coefficient of x^2 in the expansion of $(1+x)^{-3}$
14. Sum to infinity the series $\frac{4}{2.4} + \frac{4.5}{2.4.6} + \frac{4.5.6}{2.4.6.8} + \cdots + \infty.$
15. Sum to infinity the series $\frac{2.4}{3.6} + \frac{2.4.6}{3.6.9} + \cdots + \infty.$
16. Show that $3 + \frac{3.5}{8} + \cdots + \infty = 4(\sqrt{8} - 1)$
17. Prove that $\dfrac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots\ldots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots\ldots\ldots} = \dfrac{e^2+1}{e^2-1}$
18. Sum to infinity the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \cdots + \infty.$
19. Expand $e^x + e^{-x}/_2$
20. Sum to infinity the series $1 + \frac{2^3}{2!} + \frac{3^3}{3!} \ldots + \infty.$
21. Sum to infinity the series $\sum_{n=0}^{\infty} 5n + 1/(2n+1)!$
22. Sum to infinity the series $1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \infty.$
23. Sum to infinity the series $\frac{1}{0!} + \frac{2^3 x}{2!} + \frac{3^3 x}{3!} \ldots + \infty.$
24. Expand $\log(1-x)$
25. $Expand\ \log\frac{(1+x)}{(1-x)}$
26. Sum to infinity the series $\frac{11.14}{10.15.20} + \frac{11.14.17}{10.15.20.25} + \infty.$
27. Show that $\log x = \frac{x-1}{x+1} + \frac{1}{2}\left(\frac{x^2-1}{(x+1)^2}\right)$
28. When n is large Show that $\left(\frac{n+1}{n-1}\right)^{n/2} = \exp(1 + \frac{1}{3n^2})$

UNIT V

1. Write down the formula for the number of dvisors and sum of divisors
2. Find the number of dvisors and sum of divisors of 360.
3. Find the smallest number with 24 divisors.
4. Find the product of all divisors of N.

5. Define Eulers functions.
6. If N=ab where a,b are prime to each other then prove that $\phi(ab) = \phi(a)\phi(b)$.
7. Find the number of integers less then 600 and prime to it.
8. Find the sum of all the numbers which are less than 500 and prime to it.
9. Define amicable number.
10. Verify that 220 and 284 are amicable numbers.
11. Find the highest power of 2 in 10!.
12. With how many zeroes does 75! End.
13. Find the remainder when 2^{1000} is divisible by 17.
14. Show that $4^{2n+1}+3^{n+2} \equiv 0$ (mod13).
15. $2^{23} \equiv 1$ (mod 47)
16. Find the remainder 47 is divisible by 19.
17. Find a number having remainder 5,4,3,2 when divided by 6,5,4,3 respectively.
18. State Fermat's theorem .
19. If p is odd prime number and n is prime to p then $n^{p-1/2} \pm 1 = 0$(mod p).
20. Prove that any square number is 5n or $5n \pm 1$.
21. If m,n are prime number $m^{n-1}+n^{m-1}-1=0$ (mod p).
22. Prove that 18!+1 =0 (mod437).

TRIGONOMETRY – CMA 12

2 MARK QUESTIONS

UNIT I

1. Write down the formula for the expansion of sinnθ and cosnθ
2. Write the expansion of tannθ
3. Solve the equation $\cos\theta = cos\,\alpha$
4. Show that for any positive integer n $sin\theta = ncos^{n-1}\theta sin\theta - \frac{n(n-1)(n-2)}{3!}cos^{n-3}\theta sin^3\theta + \cdots$.
5. Write the expansion of $\frac{sin4\theta}{sin\theta}$
6. Prove that $cos6\theta = 1 - 18sin^2\theta + 48sin^4\theta - 32sin^6\theta$
7. Write the expansion of cos4θ
8. Write down the formula for tan(A+B+C…)
9. Express sin3θ interms of sinθ
10. Solve the equation $\sin\theta = sin\,\alpha$

UNIT II

1. Show that $tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15}$ upto five terms
2. Show that the error involved in replacing $\frac{1}{6}$ (8sinθ$-sin2\theta$) by θ is approximately $\frac{1}{30}\theta^5$ if θ is small
3. Evaluate $\lim_{x\to 0}\frac{tanx-sinx}{sin^3x}$
4. Prove that $16sin^5\theta = sin5\theta + 5sin3\theta + 10sin\theta$
5. If $\frac{\sin\theta}{\theta} = \frac{5045}{5046}$ prove that $\theta = 1°58'$ approximately
6. Find $\lim_{x\to 0}\frac{sin2x-2sinx}{x^3}$
7. Prove that $2^3cos^4\theta = cos4\theta + 4cos2\theta + 3$
8. The expansion of sinθ $and\ cos\theta$
9. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ show that x+y+z=xyz
10. Show that 2$\tan^{-1}x = \tan^{-1}(\frac{2x}{1-x^2})$

UNIT III

1. Show that tanh2x =$\frac{2tanhx}{1+tanh^2x}$
2. Prove that $\cosh^{-1} x = \log(x + \sqrt{x^2} - 1)$
3. Show that $cosh2x = cosh^2x + sinh^2x$
4. If x+iy=sin(A+iB) prove that (i) $\frac{x^2}{cosh^2B} + \frac{y^2}{sinh^2B} = 1$ (ii) $\frac{x^2}{sinh^2A} - \frac{y^2}{cosh^2B} = 1$
5. Prove that cosh(x+y)=coshxcoshy+sinhxsinhy
6. Show that $\tanh^{-1} x = \frac{1}{2}\log_e \frac{1+x}{1-x}$
7. Prove that $\sinh^{-1} x = \log(x + \sqrt{x^2} + 1)$
8. Show that $cosh^2x + sinh^2x$ = 1
9. Prove that cosh2x =$\frac{1+tanh^2x}{1-tanh^2x}$
10. What is the addition formula for tanh(x+y)

UNIT IV

1. Find the value of log(4+3i)
2. Show that log(1+itan α) = log sec $\alpha + i\alpha$
3. If $i^{x+iy} = A + iB\ prove\ that\ A^2 + B^2 = e^{-\pi y}$
4. Find Log(1+i)
5. Write cote's properties of the circle
6. Log(1-i)
7. Write Demovire's property
8. Resolve into factors $x^{15} - 1 = 0$
9. Obtain the expression for log i
10. Find log $\sqrt[3]{i}$

UNIT V

1. State Euler's series
2. Prove that $\pi = 2\sqrt{3}(1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} \ldots\ldots \infty)$
3. Write down the formula for the sum sin α+sin$(\alpha + 2\,\beta) + \cdots + \sin[\alpha + (n-1)\beta]$
4. Find the sum of the series $cos^2x + cos^2(x+y) + cos^2(x+2y) + \cdots$ upto n terms

5. Find the sum of the series $cosec\theta + \ cosec2\theta + cosec2^2\theta + \cdots + cosec2^{n-1}\theta$
6. State Gregory's series
7. Show that $\frac{\pi}{2\sqrt{2}} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots \dots \infty$
8. Show that $\tan^{-1}\frac{1}{2}$+$\tan^{-1}\frac{1}{3} = \frac{\pi}{4}$
9. Write the sum of cosα+cos$(\alpha + 2\ \beta) + \cdots + \cos[\alpha + (n-1)\beta]$
10. Write the series of $\tan^{-1} x$

5 MARK QUESTIONS

UNIT I

1. Prove that $cos6\theta = 32sin^6\theta - 48sin^4\theta + 18cos^2\theta - 1$
2. Find the equation whose roots are tan$\frac{\pi}{16}$, tan$\frac{5\pi}{16}$, tan$\frac{9\pi}{16}$, tan$\frac{13\pi}{16}$
3. Prove that $\frac{sin6\theta}{sin\theta} = 32cos^5\theta - 32cos^3\theta + 6cos\theta$
4. Expand $\frac{sin7\theta}{sin\theta}$ in the powers of sinθ
5. $If\ \frac{\tan\theta}{\theta}$ = $\frac{2524}{2523}$ find θ approximately
6. If α, β, γ be the roots of the equation x^3+$px^2 + qx + p = 0$ prove that $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$ radians expect when q=1
7. Derive the formula for tan(A+B+C…)
8. Prove that when $x = 2cos\theta$,$\frac{1+cos7\theta}{1+cos\theta}$=$(x^3 - x^2 - 2x + 1)^2$
9. Write down the expansion of $cos9\theta$
10. Prove that equation ahsecθ–bhcosecθ=$a^2 - b^2$has four roots and that the sum of the four values of θ which satisfy it is qual to an odd multiple of πradians

UNIT II

1. Determine a,b,c such that $\lim_{\theta \to 0} \frac{\theta(a+bcos\theta)-csin\theta}{\theta^5} = 1$
2. Expand $cos^2\theta.sin^4\theta$ in a series of cosines of multiple of θ
3. Expand $sin^6\theta$ in series of cosines of multiples of θ
4. If $cos^2\theta = Acos\theta + Bcos3\theta + Ccos5\theta$ show that the value of A, B, C
5. Find $\lim_{x \to \frac{\pi}{2}} \frac{sinx+cosx}{cos^2x}$

6. If Sin$(\frac{\pi}{6}+\theta) = 0.51$ find θ approximately
7. Prove that $sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}$ ...
8. Prove that $cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}$ ...
9. Determine a and b such that $\lim_{\theta\to 0} \frac{a-\theta sin\theta - bcos\theta}{\theta^4} = \frac{1}{12}$
10. Show that $2\tan^{-1}x = \sin^{-1}(\frac{2x}{1+x^2})$

UNIT III

1. If u+iv=cos(x+iy) prove that (i) $\frac{u^2}{cosh^2y} + \frac{v^2}{sinh^2y} = 1$ (ii) $\frac{u^2}{cos^2x} - \frac{v^2}{sin^2x} = 1$
2. If coshu=sec θ show that u=tan$(\frac{\pi}{4}+\frac{\theta}{2})$
3. Expand $cosh^6\theta$ interms of hyperbolic cosines of θ
4. If $cos\alpha.cosh\beta = cos\varphi$, $sin\alpha.sinh\beta = sin\varphi$ prove that $sin\varphi = \pm sin^2\alpha = \pm sinh^2\beta$
5. If $tanh\frac{x}{2} = tan\frac{x}{2}$ prove that cosx.coshx=1
6. If x+iy=cos(u+iv) where x,y,u,v are real prove that (i) $(1+x)^2 + y^2 = (\cosh v + cosu)^2$

(ii)$(1-x)^2 + y^2 = (\cosh v - cosu)^2$

7. Show that $\frac{1+tanhx}{1-tanhx} = cosh2x + sinh2x$
8. If tan$(\theta + i\varphi) = cos\alpha + isin\alpha$, prove that $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$ and $\varphi = \frac{1}{2}logtan(\frac{\pi}{4}+\frac{\alpha}{2})$
9. If tan(x+iy) = u+iv prove that $\frac{u}{v} = \frac{sin2x}{sinhy}$
10. If x+iy=sin(A+iB) prove that (i) $\frac{x^2}{cosh^2B} + \frac{y^2}{sinh^2B} = 1$ (ii) $\frac{x^2}{sin^2A} - \frac{y^2}{cos^2A} = 1$

UNIT IV

1. Resolve into factors the expression $x^n - a^n = 0$ if n is even
2. If $i^{i^{\cdot^{\cdot^{\infty}}}} = A + iB$ then prove that (i) $\tan\frac{\pi A}{2} = \frac{B}{A}$ $(ii) A^2 + B^2 = e^{-\pi B}$
3. Deduce the expansion of $\tan^{-1}x$ in powers of x from the expansion of log(a+ib)
4. Resolve into factors the expression $x^n + a^n = 0$ if n is odd
5. Derive cote's property of the circle
6. Reduce $(\alpha + i\beta)^{x+iy}$ in the form A+iB

7. Find the value of log$\left(\frac{1+cos\theta+isin\theta}{cos\theta-1+isin\theta}\right)$
8. If tanlog(x+iy) = a+ib where $a^2+b^2\neq 1$ show that tanlog$(x^2+y^2)=\frac{2a}{1-a^2-b^2}$
9. If $e^{iA}=i^B$ then $\frac{A}{B}$= 2n$\pi+\frac{\pi}{2}$
10. $i^i=e^{-(4n+1)\frac{\pi}{2}}$ where n is an integer

UNIT V

1. Sum to n terms $\tan^{-1}\frac{1}{1+1+1^2}+\tan^{-1}\frac{1}{1+2+2^2}+\tan^{-1}\frac{1}{1+3+3^2}$ ....
2. Sum the series $sin^3\frac{\theta}{3}+3sin^3\frac{\theta}{3^2}$+$3^2sin^3\frac{\theta}{3^3}+\cdots to\ n\ terms$
3. Sum the series sinhx+sinh(x+y)+ sinh(x+2y)+… n terms
4. Find the sum to n terms of the series cosecθcosec2$\theta+cosec2\theta cosec3\theta+cosec3\theta cosec4\theta+\cdots$
5. Find the sum of the series $\tan^{-1}\frac{x}{1+1.2^2}+\tan^{-1}\frac{x}{1+2.3^2}+\ldots+\tan^{-1}\frac{x}{1+n(n+1)x^2}$
6. Find the sum of the series sin α+sin$(\alpha+2\,\beta)+\cdots+\sin[\alpha+(n-1)\beta]$
7. Sum to n terms $\frac{sin\theta}{sin2\theta sin3\theta}$+$\frac{sin\theta}{sin3\theta sin4\theta}+\frac{sin\theta}{sin4\theta sin5\theta}+\cdots$
8. Sum the series tanθsec2$\theta+tan2\theta sec4\theta+\cdots+tan2^{n-1}\theta sec2^n\theta$
9. Sum the series $sec\alpha sec3\alpha+sec2\alpha sec4\alpha+\cdots$
10. Sum to infinity the series $csin\alpha+\frac{c^2}{2!}sin2\alpha+\frac{c^3}{3!}sin3\alpha+\cdots$.

10 MARK QUESTIONS

UNIT I

1. Find the equation whose roots are 2cos$\frac{2\pi}{7}$,2cos$\frac{4\pi}{7}$,2cos$\frac{6\pi}{7}$
2. Show that cos$\frac{2\pi}{9}$. cos$\frac{4\pi}{9}$ .cos$\frac{6\pi}{9}$. cos $\frac{8\pi}{9}$ =$\frac{1}{16}$
3. Prove that $cos8\theta=1-32sin^2\theta+160sin^4\theta-256sin^6\theta+128sin^8\theta$
4. Show that the equation $\sin(\theta+\alpha)=asin2\theta+b$ has four roots and that if they are $\theta_1,\theta_2,\theta_3,\theta_4$then $\theta_1+\theta_2+\theta_3+\theta_4=(2k+1)\pi$
5. Prove that $\frac{sin9\theta}{sin\theta}=256cos^8\theta-448cos^6\theta+240cos^4\theta-49cos^2\theta+1$
6. Prove that sin$\frac{\pi}{5}$. sin$\frac{2\pi}{5}$ .sin$\frac{3\pi}{5}$.sin$\frac{4\pi}{5}$=$\frac{5}{16}$

7. Prove that the equation cos2 $\theta + acos\theta + bsin\theta + c = 0$ has in general four solution $\alpha, \beta, \gamma, \delta\ \ lying\ \ between\ 0\ and\ \ 2\pi\ and\ \alpha + \beta + \gamma + \delta$ is a multiple of π
8. Expand sin7θ as a polynomial in sin θ.Hence obtain the cubic equation whose roots are $sin^2\frac{2\pi}{7}, sin^2\frac{4\pi}{7}, sin^2\frac{6\pi}{7}$
9. Expand tan4θ in terms of tanθ and show that tan$\frac{\pi}{16}$, tan$\frac{5\pi}{16}$, tan$\frac{9\pi}{16}$, tan$\frac{13\pi}{16}$ are roots of the equation
10. Derived expansion of $cosn\theta and\ sinn\theta$

UNIT II

1. Expand $cos^5\theta.sin^3\theta$ in a series of sines of multiple of θ
2. Solve approximately in radians sin($\frac{\pi}{3} + x) = 0.87$
3. Show that $cos^2\theta.sin^6\theta = -\frac{1}{2^7}(cos8\theta - 4cos6\theta + 4cos4\theta + 4cos2\theta - 5)$
4. Prove that$2^7sin^8\theta = cos8\theta - 8cos6\theta + 28cos4\theta - 56cos2\theta + 35$
5. Find $\lim_{x\to 0}\frac{tanx - sinx}{sin^3x}$
6. Prove that $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$
7. Prove that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$
8. Prove that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$
9. Solve the equation $\tan^{-1}2x + \tan^{-1}3x = \frac{3\pi}{4}$
10. Prove that $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$

UNIT III

1. If tan(A+iB)= x+iy prove that (i) $x^2 + y^2 + 2xcot2A = 1$

   $(ii)x^2 + y^2 - 2ycoth2B = -1$
2. Prove that $\tanh^{-1}x = \frac{1}{2}\log(\frac{1+x}{1-x})$
3. If log sin$(\theta + i\varphi) = A + iB$ prove that (i) $cos(\theta - B) = e^{2\varphi}cos(\theta + B)$

   (ii) cosh2$\varphi - cos2\theta = 2e^{2A}$
4. Prove that $\frac{coshx-1}{sinhx} = \frac{sinhx}{coshx+1} = tanh\frac{x}{2}$

5. If cos(x+iy) = r($\cos\alpha + i\sin\alpha$) show that $y = \frac{1}{2}\log[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}]$

6. Separate into real and imaginary part of $\tan^{-1}(x+iy)$

7. Expand $cosh^8\theta$ interms of hyperbolic cosines of θ

8. If $\cos^{-1}(\alpha + i\beta) = \theta + i\varphi$ show that (a) $\alpha^2 sech^2\varphi + \beta^2 cosech^2\varphi = 1$

(b)$\alpha^2 sec^2\theta - \beta^2 cosec^2\theta = 1$

9. If$u = logtan(\frac{\pi}{4} + \frac{\theta}{2})$show that (i) $\theta = -ilogtan\left(\frac{\pi}{4} + i\frac{\pi}{2}\right)$

(ii) tanh$\frac{u}{2} = tan\frac{\theta}{2}$

10. Separate into real and imaginary part of $tanh(1+i)$

UNIT IV

1. Resolve into factors the expression $x^{2n} - 2x^n a^n cosn\theta + a^{2n}$. Deduce

   (i) $\frac{x^n - a^n cosn\theta}{x^{2n} - 2x^n a^n cosn\theta + a^{2n}} = \frac{1}{nx^{n-1}}\sum_{r=0}^{n-1}\frac{x - a\cos(\theta + \frac{2r\pi}{n})}{x^2 - 2x\, a\cos\left(\theta + \frac{2r\pi}{n}\right) + a^2}$

   (ii) $If\ x = cos\alpha + isin\alpha$ show that cos$n\alpha - cosn\theta = 2^{n-1}\prod_{r=0}^{n-1}[cos\alpha - \cos(\theta + \frac{2r\pi}{n})]$

2. Resolve into factors the expression $x^n + a^n = 0$

3. Resolve into factors the expression $x^n - a^n = 0$

4. If log sin$(\theta + i\varphi) = L + iB$ prove that (i) $cos(\theta - B) = e^{2\varphi}cos(\theta + B)$

(ii) cosh$2\varphi - cos2\theta = 2e^{2L}$

5. If tan$(\theta + i\varphi) = cos\alpha + isin\alpha$ show that (i) $\theta = \frac{n\pi}{2} + \frac{\pi}{4}(i)\ \varphi = \frac{1}{2}logtan(\frac{\pi}{4} + \frac{\alpha}{2})$

6. Find the real factors of $x^7 - 1$ andcos$\frac{2\pi}{7}$ + cos$\frac{4\pi}{7}$ +cos$\frac{6\pi}{7}$ = - $\frac{1}{2}$

7. If $i^i = x + iy$ considering only principal terms value show that (i) $y = x\ tan\frac{\pi x}{2}$

(ii)$x^2 + y^2 = e^{-\pi y}$

8. If log log log$(\alpha + i\beta) = p + iq$ $(i)\frac{1}{2}$log$(\alpha^2 + \beta^2) = e^{pcosq}\cos(e^p sinq)$

(ii) $\tan^{-1}\frac{\beta}{\alpha} = e^{pcosq}\sin(e^p sinq)$

9. Express $\tan^{-1}(cos\theta + isin\theta)$ in the form of A+iB

10. Resolve into factors $x^{2n} - 2x^n a^n cosn\theta + a^{2n}$ and deduce sinn$\alpha = 2^{n-1} \prod_{r=0}^{n-1} \sin(\alpha + \frac{r\pi}{n})$

UNIT V

1. Sum the series upto n terms (i) sin α+sin2α + sin 3α ...

   (ii)cosα+$\frac{1}{2}$cos($\alpha + \beta$) + $\frac{1.3}{2.4}$cos($\alpha + 2\beta$) ...

2. Sum upto n terms $sin^3\alpha + sin^3 2\alpha + sin^3 3\alpha + \cdots$

3. Find the sum of the series $ccos\alpha - \frac{c^3}{3}cos3\alpha + \frac{c^5}{5}cos5\alpha$ ....

4. Sum the series sinα+csin($\alpha + \beta$) + c^2sin($\alpha + 2\beta$) ...

5. Sum the series $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21} + \cdots$

6. Sum the series (i) $cos\theta - \frac{1}{2}cos2\theta + \frac{1}{3}cos3\theta \ldots \infty$

   (ii) $sin\theta - \frac{1}{2}sin2\theta + \frac{1}{3}sin3\theta \ldots \infty$

7. Sum the series (i) $cos\theta - \frac{1}{3}cos3\theta + \frac{1}{5}cos5\theta \ldots \infty$

   (ii) $sin\theta - \frac{1}{3}sin3\theta + \frac{1}{5}sin5\theta \ldots \infty$

8. Find the sum of the series cos α+cos($\alpha + 2\beta$) + $\cdots$ + cos[$\alpha + (n-1)\beta$]

9. Prove that $\frac{\pi}{4} = (\frac{2}{3} + \frac{1}{7}) - \frac{1}{3}(\frac{2}{3^3} + \frac{1}{7^3}) + \frac{1}{5}(\frac{2}{3^5} + \frac{1}{7^5}) \ldots$

10. Sum to infinity (i) sin$\alpha + \frac{sin2\alpha}{2!} + \frac{sin3\alpha}{3!} + \cdots \infty$, (ii) 1+cos$\alpha + \frac{cos2\alpha}{2!} + \frac{cos3\alpha}{3!} + \cdots \infty$

Mathematical Statistics-I – CAMA 13B

Section-A

1. State Baye’s theorem.
2. Define : Independent events
3. Define probability.
4. Define probability Density Function.
5. Define distribution functions.
6. Define expectation.
7. Define MGF.
8. Write the properties of characteristic function.
9. State uniqueness theorem of characteristic function.
10. Write Pearson formula for the rank correlation coefficient.
11. Define: Line of Regression.
12. State any two properties of regression coefficients.
13. Define Uniform Distribution.
14. Define Bernolli Distribution.
15. Define sample space.
16. What is an event?
17. Define expectation.
18. Define cumulant generating function.
19. Define characteristic function.
20. State inversion theorem.
21. Define correlation.
22. Define Regression.
23. Write the formula for rank correlation .
24. Define Poisson distribution.
25. Define Binomial distribution.
26. Define continuous distribution.
27. Define discrete distribution.
28. Define Normal distribution.
29. Define event?
30. If A and B are independent events with P(A)=5/6, P(B)=6/7 then the value of $P(A\cap B)$.
31. Find the expectation of the number on a die?
32. Define Random variable?
33. What is negative correlation?
34. What is positive correlation?
35. Write the bounds of correlation coefficient
36. If 0.4 and 0.9 is regression coefficient then finding the correlation coefficient?
37. Write the probability density function of exponential distribution.

38. Write probability density function of binomial distribution.
39. Write MGF of binomial distribution.
40. Write MGF of Normal distribution.
41. Comment on the following: In a Poisson distribution mean is 5 and variance is 25.
42. What is Regression formula X on Y and Y on X.
43. Write the characteristic function of Geometric distribution.
44. Write MGF of Geometric distribution.

Section-B

1. For any two events A and B prove that p (A∪ B) =p (A) +p (B)-p (A∩ B).
2. Prove that P ($\bar{A}$) =1-P(A).
3. Show that E(xy)=E(x).E(y).
4. Write down the axioms of Probability.
5. Two unbiased dice are thrown , find the probability (i) both show the same number (ii) the first die show 6 (iii) the total of number of die is 8 (iv) the total of number of die is 13 (v) the total of no of die any from 2 to 12 ?
6. From a city population, the probability of selecting (i) a male or a smoker is 7/10, (ii) a male smoker is 2/5, and (iii) a male if a smoker is already IS 2/3. Find the probability of selecting a) a non-smoker b) a male c) a smoker, if a male is first selected ?
7. A continuous random variable X has following probability density function

   f(x)=$\begin{cases} K(x-x^2), 0 \le x \le 1 \\ 0 \quad\quad otherwise. \end{cases}$ Find the value of K and mean of X.
8. Find the m.g.f of the random variable whose moments are $\mu_r = (r+1)!\,2^r$
9. Write the properties of characteristic function.
10. State and prove uniqueness theorem of characteristics functions.
11. Show that$^{\phi}{}_{x_1x_2}(t_1, t_2) = {}^{\phi}{}_{x_1}(t_1)^{\phi}{}_2(t_2)$.
12. Find rank correlation for the following data:

Rank 1	3	5	8	4	7	10	2	1	6	9
Rank2	6	4	9	8	1	2	3	10	5	7

13. Prove that correlation coefficient is the geometric mean between two regression coefficients.
14. Drive moment generating function for binomial distribution.
15. Drive MGF for Poisson distribution.
16. Drive PDF for Normal distribution.

17. Drive PDF for binomial distribution.

18. Write any five properties of normal distribution.

19. Write any five properties of Geometric distribution

Section-C

1. State Addition and Multiplication theorem of Probability.
2. For any two events A, B prove that $\mathrm{P}(\mathrm{A}/B) = \frac{\mathrm{P(A \cap B)}}{P(B)}$ if P(B)>0 and $\mathrm{P}(B/A) = \frac{\mathrm{P(A \cap B)}}{P(A)}$ if P(A)>0.
3. If f(x,y) = $\begin{cases} \frac{1}{8}\ (6 - x - y) 0 < x < 2, 2 < y < 4 \\ 0 \qquad\qquad otherwise \end{cases}$
4. State and Prove Bayes theorem.
5. A bag contains 'a' white balls and 'b'black balls. If 'c' balls drawn at random then find the expected value of the numbers of white balls drawn.
6. State and prove Chebychev's inequality.
7. Calculate Karl-Pearson co-efficient of correlation for the following data:

X	6	7	9	8	4	7	9	3	1
Y	15	16	14	12	15	17	8	9	10

8.calculate correlation coefficient:

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

9.Find moment generating function for normal distribution about origion.

10.Find the MGF for Binomial distribution about origion.

11.Calculate spearman's rank correlation

X	80	78	75	56	68	60	59	60
Y	12	13	14	14	15	27	16	12

12.Find mean and variance of Poisson distribution.

13.Find mean and variance of Binomial distribution.

14.Find mean and variance of normal distribution.

15.Find mean and variance of Geometric distribution.

16. $If\ \ X \sim N(\mu, \sigma^2)\ with\ \mu = 30, \sigma = 5\ find:$
$i)\ P(26 \leq X \leq 40), ii)\ P(X \geq 45) iii)\ P(|x - 30| > 5)$

17. Calculate the coefficient of correlation and obtain the line of regression for the following data

x	1	2	3	4	5	6	7
y	9	8	10	11	12	13	14

18. Find moment generating function for normal distribution about origin.

II B.Sc., MATHEMATICS

SEMESTER III

DIFFERENTIAL EQUATIONS - CMA 31

2-MARK QUESTIONS

UNIT-I ORDINARY LINEAR DIFFERENTIAL EQUATIONS

1. Find whether the equation $(2y^2 - 4x + 5)dx - (4 - 2y + 4xy)dy$ is exact or not.
2. Find the general solution of the equation$y = xp + p^2$.
3. Solve $p^2 - 5p + 6 = 0$, where p=dy/dx.
4. Solve y=px+a/p.
5. Prove that the equation (x+2/y)dy+ydx=0 is exact.
6. Solve y=(x-a)p-p^2.
7. Solve $p^2 + 3p + 2 = 0$, where p=dy/dx.
8. Solve$y = px - \frac{1}{p^2}$.
9. Define exact differential equation.
10. Find the general solution of $y = xp + \frac{a}{p}$.

UNIT-II ORDINARY LINEAR DIFFERENTIZL EQUATIONS [CONTINUATION]

11. Write the Cauchy's linear equation.
12. Solve the equation $x^3 \frac{d^3y}{dx^3} - 6y = 0$.
13. Solve y"+16y=0.
14. Write a general linear non homogeneous equation of second order with variable coefficients.
15. Solve $(D^2 + 2D + 5)y = 0$.
16. Solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = 0$
17. Solve y"+6y'+9y=0.
18. Write a general linear homogeneous equations of n-th order with constant coefficients.
19. Find the P.I of $(D^2 + 5D + 6)y = e^x$.
20. Find the C.F of $(D - 2)^2 y = 8(e^{2x} + sin2x + x^2)$.

UNIT-III DIFFERENTIAL EQUATIONS OF OTHER TYPES

21. Verify the condition of integrability of the equation, (y+z)dx+(z+x)dy+(x+y)dz=0.
22. Solve dx/yz=dy/xy=dz/xy.
23. What is the necessary condition for the integrability of the equation pdx+Qdy+Rdz=0 where P,Q,R function of x,y,z.
24. Verify that the equation $(3x^2)(y + z)dx + (z^2 + x^3)dy + 2yz + x^3)dz = 0$ is exact.
25. Solve $\frac{d^2y}{dx^2} = xe^x$.
26. Verify the condition of integrability of the equation, $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$.
27. Solve xdx+zdy+(y+2z)dz=0.
28. Solve yzdx+zxdy+xydz=0.
29. Solve yzdx+2zxdy-3xydz=0

30. Solve $\frac{xdx}{y^2z} = \frac{dy}{zx} + \frac{dz}{y^2}$.

UNIT-IV LAPLACE TRANSFORM

31. Find L(sin^2t)

32. Find$L^{-1}[\frac{1}{s(s-2)}]$.

33. Find L(cos4tsin2t)

34. Find$L^{-1}[\frac{1}{(s+1)^2}]$.

35. Find L($t^2 - 3t + 2$).

36. Find $L^{-1}[\frac{1}{(s+3)^6}]$

37. Find L[$sin^2 2t$]

38. Find $L^{-1}\left[\frac{1}{(s+a)^2}\right]$

39. Find L[$cos^2 2t$]

40. Find $L^{-1}\left[\frac{s^2-3s+4}{s^3}\right]$

UNIT-V PARTIAL DIFFERENTIAL EQUATIONS

41. Find the partial differential equation by eliminating the arbitrary constants from $Z = ax^3 + by^3$.

42. Eliminating the arbitrary functions f and g and from the partial differential equation Z=f(x).g(y).

43. From the partial differential equation by eliminating the constants a,b in $Z = (x-a)^2 + (y-b)^2 + 1$.

44. Solve the equation $\frac{\partial z}{\partial y} = sinx$.

45. From the partial differential equation by eliminating the arbitrary constants from $Z = (x^2 + a)(y^2 + b)$

46. From the partial differential equation by eliminating the arbitrary function from $Z = f(x^2 + y^2)$

47. Solve $\frac{\partial z}{\partial x} = 0$.

48. Solve pq=k.

49. Form the partial differential equation from the equation $2Z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

50. Solve $\sqrt{p} + \sqrt{q} = 1$.

5 MARK QUESTIONS

UNIT-I

1. Solve y(2xy+e^x)dx=e^xdy.
2. Solve p=sin(y-xp) Also find the singular solution.
3. Solve dy/dx-dx/dy=x/y-y/x.
4. Solve x=y+alogp.
5. Solve $\frac{dy}{dx} - ytanx = sinxcos^2x/y^2$.
6. Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.
7. Solve $y = x + p^2 - 2p$.

8. Solve $x = p + p^4$.
9. Solve $\frac{dy}{dx} + ytanx = cosx$.
10. Solve y=psinp+cosp.

UNIT-II

11. Solve $\frac{d^2y}{dx^2} - 4y = xsinhx$.
12. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = logx$.
13. Solve $\frac{d^2y}{dx^2} + 4y = 1 + x^2$
14. Solve $\frac{d^2y}{dx^2} + y = cosecx$
15. Solve $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3logx$.
16. Solve the equation $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$.
17. Solve $y'' - 6y' + 9y = e^x$.
18. Solve $y'' - 62 + 2y = e^x tanx$.
19. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} = x + 1$.
20. Solve $(D^2 - 5D + 6)y = sin3x$.

UNIT-III

21. Solve $x\frac{d^2y}{dx^2} = \sqrt{1 + (\frac{dy}{dx})^2}$.
22. Solve $\frac{dx}{x(y^2-z^2)} = \frac{dy}{-y(z^2+x^2)} = \frac{dz}{z(x^2+y^2)}$.
23. Solve zydx=zxdy+y^2dz
24. Solve(yz+xyz)dx+(zx+xyz)dy+(xy+xyz)dz=0
25. $\frac{dx}{y-xz} = \frac{dy}{yz+x} = \frac{dz}{x^2+y^2}$.
26. Solve $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$.
27. Solve yzdx-2zydy-3xydz=0.
28. Solve xdx+zdy+(y+2z)dz=0.
29. Solve $2x^3y + 1)dx + x^4dy + x^2tanzdz = 0$.
30. Solve $\frac{d^3y}{dx^3} = xe^x$.

UNIT-IV

31. Find L[$\frac{1-e^t}{t}$]
32. Find $L^{-1}[\frac{2s^2-6s+5}{s^3-6s^2+11-6}]$
33. Find L[te$^t sint$]
34. Find$L^{-1}[\frac{s-5}{s^2-3s+2}]$
35. Find $L^{-1}[log\frac{1+s}{s-2}]$

36. FindL^{-1}[log$\frac{1}{(s+1)(s+3)}$]
37. Find L$[e^{-t}cosht]$
38. Find $L^{-1}[\frac{1}{s^2+2s+2}]$
39. Find L$[t^2sin2t]$
40. Find $L^{-1}[\frac{s}{(s+2)^2}]$

UNIT-V

41. Solve $p^2+q^2=x+y$
42. Solve pz=1+q^2
43. Solve (1-x)p+(2-y)q=3-z
44. Solve p$e^y=qe^x$
45. Solve the equation $p^2+q^2=4$
46. Solve $p^2+q^2=18$
47. Solve pq=x
48. Solve p+q=x+y
49. Solve z=px+qy+$\frac{p}{q}$
50. Solve $z^2(p^2+q^2)=x^2+y^2$
51. Solve $x^2p^2+y^2q^{2=}z^2$

10 MARK QUESTIONS

UNIT-I

1. Solve $xy(1+xy^2)\frac{dy}{dx}=1$
2. Solve $p^2+2pycotx=y^2$
3. Solve the equation $xp^2+2px-y=0$ (p=dy/dx)
4. Solve (a) sinpxcosy=cospxsiny+p (b) Solve $y=2p+3p^2$
5. Solve $y=3px+6p^2y^2$
6. Solve $[\frac{2}{\sqrt{1-x^2}}+ycos(xy)]dx+[xcosxy-y^{\frac{-1}{3}}]dy=0$
7. Solve $(x^3y^3+x^2y^2+xy+1)ydx+(x^3y^3-x^2y^2-xy+1)xdy=0$
8. Solve $y-2px-tan^{-1}(xp^2)$
9. Solve $e^{3x}(p-1)+p^3e^{2y}=0$
10. Solve $p=tan^{-1}[x-\frac{p}{1+p^2}]$

UNIT-II

11. Solve the simultaneous equation $\frac{dx}{dt}+5x-2y=t;\frac{dy}{dt}+2x+y=0$,given that x=y=0 when t=o.
12. Solve $y^{"}+9y=cos3x$
13. Solve $(D^{2+}2D+5)y=e^{-x}tanx$[using method of variation of parameters]
14. Solve $\frac{dx}{dt}+7x+y=0;\frac{dy}{dt}$+2x+5y=0

15. Solve $\frac{d^2y}{dx^2} + 4y = 1 + x^2$
16. Solve (D^2-4D-12)y=sinxsin2x
17. Solve $\frac{d^2y}{dx^2} + 4y = xsinx$
18. Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(logx)$
19. Solve $x\frac{d^2y}{dx^2} - \frac{2y}{x} = x + \frac{1}{x^2}$
20. Solve $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

UNIT-III

21. Solve $(y^2 + yz)dx + z^2 + zx)dy + (y^2 - xy)dz = 0$
22. Show that $(x^2y - y^3 - y^2z)dx + (xy^2 - x^2z - x^3)dy + (xy^2 + x^2y)dz = 0$
23. Solve $2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$
24. Solve $y\frac{d^2y}{dx^2} - (\frac{dy}{dx})^2 = y^2logy$
25. Solve $\frac{d^2y}{dx^2} = 3\sqrt{y}$. Given that y=1, $\frac{dy}{dx}$=2, where x=0
26. Solve (mz-ny)dx+(nx-lz)dy+(ly-mx)dz=0
27. Show that $(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0$
28. Solve $y\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{dy}{dx} - 2y\right) = 0$
29. (a) Solve $(x+z)^2dy + y^2(dx + dz) = 0$; (b) Solve $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$
30. (a) Solve $\frac{dx}{3z-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$; (b) Solve $x\frac{d^2y}{dx^2} = \sqrt{1 + (\frac{dy}{dx})^2}$

UNIT-IV

31. Using Laplace transform, Solve: $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$, given that y(0)=1,y'(0)=0, y''(0)=-2.
32. Using Laplace transform Solve: $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y = 2, y(O) = 1, y'(0) = 1.$
33. Solve $y" - 3y' + 2y = e^{2t}\ , y(o) = -3, y, (0) = 5$ using Laplace transform.
34. Solve the equation using Laplace transform $\frac{dx}{dt} + \frac{dy}{dt} = -2sint, \frac{dx}{dt} - \frac{dy}{dt} = 2cost, x(0) = y(0) =$ 1.
35. Using Laplace transform solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dx} - 3y = sint$. Given that y=0 and $\frac{dy}{dt}$=0 when t=0.
36. If L[f(t)]=$\bar{f}(s)$. Show that
    (a) L[sinhatf(t)]=1/2[$\bar{f}$(s-a)-$\bar{f}(s+a)$]
    (b) L[coshatf(t)]=1/2[$\bar{f}(s-a) + \bar{f}(s+a)$] Hence evaluate (i) sinh2tsin3t (ii) cosh3tcos2t.
37. If f(t) is a periodic function with period T (ie) f(t+T) = f(t) then L{f(t)}=$\frac{\int_0^T e^{-st}f(t)dt}{1-e^{-sT}}$.
38. Applying convolution theorem evaluate $\frac{1}{s^3(s^2+1)}$.
39. Find $L^{-1}[tan^{-1}\frac{2}{s^2}]$.

40. Find $L^{-1}[\frac{s+2}{(s^2+4s+8)^2}]$.

UNIT-V

41. Solve: $(x^2 - yz)p + y^2 - zx)q = z^2 - xy$.
42. Solve pg=y.
43. Solve (a) $p^2 + pq = z^2$.
    (b)$q^2 - p = y - x$.

44. Solve $x^2 + y^2 = p^2 - q^2$.

45. Solve $p(1 + q)^2 = q(z - 1)$.

46. Eliminating the arbitrary function from the following:

    (a) f(x+y+z,xyz)=0
    (b) f(z/x,y/x)=0

47. Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

48. Find the complete and singular solution of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.

49. (a) Define complete solution.

    (b) Find the complete and singular solution of $z = p^2 + q^2$.

50. (a) Define singular solution

    (b) Solve $z^2(p^2 + q^2) = x^2 + y^2$

Numerical Methods – I – CAMA 13A

Unit – I Finite Difference

2 Marks

1. Define Forward difference formula.
2. Write a general form for a forward difference formula?
3. Define Leading terms and Leading differences.
4. Define Backward difference formula.
5. Writa a general form for a backward difference formula?
6. Write an Indices law for the operator Δ?
7. Prove that $\Delta (f(x) . g(x)) = f(x+h) \Delta g(x) + g(x) \Delta f(x)$.
8. Prove that $\Delta[\frac{f(x)}{g(x)}] = \frac{g(x) \Delta f(x) - f(x) \Delta g(x)}{g(x+h) g(x)}$.
9. Define Shifting operator E.
10. Prove that the operator E and Δ are distributive.
11. Show that $E[c f(x)] = c E f(x)$ and $\Delta[cf(x)] = c \Delta f(x)$.
12. Prove that $E^m . E^n = E^{m+n} f(x)$ and $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$.
13. Prove that $E\nabla = \nabla E = \Delta$.
14. Evaluate Δe^x.
15. Evaluate $\Delta \log x$.
16. Evaluate $\Delta \sin(ax+b)$.
17. Evaluate $\Delta \cos(ax+b)$.
18. Evaluate $\Delta \tan(ax+b)$.
19. Evaluate $\Delta = E\nabla$.
20. Evaluate $E^{-1} = 1 - \nabla$.
21. Evaluate $\Delta^r y_k = \nabla^r y_{k+r}$.
22. Evaluate$\nabla = E^{-1} \Delta$.
23. Evaluate$\nabla^r y_k = \Delta^r y_{k-r}$.
24. Evaluate $E = e^{hD} = 1+ \Delta$.
25. Evaluate$\nabla = 1 - e^{-hD}$.
26. Evaluate $E\Delta = \Delta E$.
27. Evaluate $\Delta \tan^{-1} x$.
28. Evaluate $\Delta \log f(x)$.
29. Evaluate $(\frac{\Delta^2}{E}) x^3$.
30. Define Factorial Polynomial.

5 Marks

1. State and Prove Fundamental theorem of difference calculus.
2. Construct a forward difference table from the following values of x and y :

x	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
y	2.60	3.00	3.40	4.28	7.08	14.20	29.00

3. Evaluate $(1 + \Delta)(1 - \nabla) = 1$.
4. Evaluate $\Delta^n x^n$.
5. Evaluate $\Delta^n e^x$.
6. Evaluate $\Delta^n(e^{ax+b})$ in interval of differencing unity.
7. Evaluate $\left(\frac{\Delta^2}{E}\right) sin\,(x + h)$.
8. Explain difference between $\left(\frac{\Delta^2}{E}\right) u_x$ and $\frac{\Delta^2 u_x}{E u_x}$. Find the values when $u_x = x^3$.
9. Prove that $\Delta x^{(n)} = nhx^{(n-1)}$.
10. Prove that $\Delta^n x^{(n)} = n!$.
11. Evaluate $\Delta^2\left(\frac{5x+12}{x^2+5x+6}\right)$.
12. Express $f(x) = x^4 – 5x^3 + 3x + 4$ in terms of factorial polynomial.
13. Obtain the function whose first difference is $9x^2 + 11x + 5$.
14. Find the second difference of the polynomial $7x^4 + 12x^3 – 6x^2 + 5x – 8$ with interval of differencing as h = 2.
15. Evaluate $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$.
16. Evaluate $\mu^{-1} = 1 - \frac{\delta^2}{8} + \frac{3}{128}\delta^4 - \frac{5}{1024}\delta^6 + \cdots$ ...
17. Find the cubic polynomial which states the following sets of values (0,1),(1,2),(2,1),(3,10).
18. Given y(75) = 246, y(80) = 202, y(85) = 118, y(90) = 40, find y(79).

10 Marks

1. Evaluate $\Delta^3[(1 - x)(1 - 2x)(1 - 3x)]$ *and*

$\Delta^{10}[(1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)]$.

2. Represent the function $f(x) = x^4 – 12x^3 + 24x^2 – 30x + 9$ and its successive difference in factorial notation.
3. Given sin45˚ = 0.7071, sin50˚ = 0.7660, sin55˚ = 0.8192, sin60˚ = 0.8660, find sin52˚.
4. Compute the value of $\sqrt{5.5}$ given that $\sqrt{5}$ = 2.236, $\sqrt{6}$ = 2.4149, $\sqrt{7}$ = 2.64, $\sqrt{8}$ = 2.828.
5. Given the table

x	0.0	0.5	1.0	1.5	2.0
f(x)	0.3989	0.3521	0.2420	0.1295	0.0540

Evaluate f(1.8).

6. Given the table

x	0.0	0.1	0.2	0.3	0.4
e^x	1	1.1052	1.2214	1.3499	1.4918

Find the value of $y = e^x$ when x = 0.38.

7. Apply Newton's backward formula for a polynomial of degree 3 which include following x,y pairs

x	3	4	5	6
y	6	24	60	120

8. The population of a town in the decennial census was as given below. Estimate the population for the years 1895 & 1925.

Year x	1891	1901	1911	1921	1931
Population y	46	66	81	93	101

9. The following are data from the steam table.

Temperature ˚C	140	150	160	170	180
Pressure kgf/cm^2	3.685	4.854	6.302	8.076	10.226

Using Newton's formula find the pressure of the steam for a temperature of 142˚.

Unit – II Central difference

2 Marks

1. Define Central difference interpolation.
2. What are the advantages of central difference interpolation formula?
3. Show that $\nabla\Delta = \nabla - \Delta = \delta^2$.
4. Show that $E = (\frac{\Delta}{\delta})^2$and $= \Delta E^{\frac{-1}{2}} = \nabla E^{\frac{1}{2}}$.
5. Write a formula for Gauss's forward interpolation formula.
6. Write a formula for Gauss's backward interpolation formula.
7. Write a formula for Sterling's interpolation formula.
8. Write a formula for Bessel's interpolation formula.
9. Write down a special case for Bessel's formula.
10. Write down a general rule for a Central difference interpolation.
11. What is the range for Sterling's formula?
12. What is the range for Bessel's formula?
13. Give the relation between Stirling's formula and Gauss formula.
14. Show that $\mu\delta = ½ (\nabla + \Delta)$.
15. Show that $\delta = 2 \sinh\frac{u}{2}$.
16. Show that $\mu\delta$ = Sinhu.
17. Show that $(E + 1)\delta = 2(E-1)\mu$.

5 Marks

1. Using Gauss forward formula, find the value of f(3.5) from the following data

x	2	3	4	5
f(x)	2.626	3.454	4.784	6.986

2. Using Gauss backward formula, find the value of $\sin 45^{\circ}$ from the following data

x	20	30	40	50	60	70
sinx	0.34202	0.502	0.64279	0.76604	0.86603	0.93969

3. Derive Sterling's formula from Gauss forward and backward formula.
4. Using Sterling's formula find y_{35} given that $y_{20} = 512$, $y_{30} = 439$, $y_{40} = 346$, $y_{50} = 243$.
5. Derive Bessel's interpolation formula.
6. Given

x	0	4	8	12
f(x)	143	158	177	199

Find f(5) using Bessel's formula.

7. Given $\sqrt{12500} = 111.8033$, $\sqrt{12510} = 111.8481$, $\sqrt{12520} = 111.8928$, $\sqrt{12530} = 111.9374$, find $\sqrt{12516}$ using Gauss backward formula.
8. Using Bessel's formula find y_{25} given that $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.
9. Using Gauss forward formula, find the value of f(3.5) from the following data

x	2	3	4	5
f(x)	2.626	3.454	4.784	6.986

10. Using Sterling's formula, find f(1.22) from the following data

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
f(x)	0.84147	0.89121	0.93204	0.96356	0.98545	0.9974 9	0.9995 7	0.9938 5	s0.97385

11. From the following table find the value of f(2.73) using Bessel's formula

x	2.5	2.6	2.7	2.8	2.9	3.0
f(x)	0.4938	0.4953	0.4965	0.4974	0.4981	0.4987

12. Interpolate by means of Gauss backward formula of interpolation the sales of a concern for the year 1966 given that

Year	1931	1941	1951	1961	1971	1981
Sales	12	15	20	27	39	52

10 Marks

13. Using Gauss forward formula, find the value of log 3375 from the following data

x	310	320	330	340	350	360
logx	2.4914	2.5051	2.5185	2.5315	2.5441	2.5563

14. Applying Gauss forward formula, fine the value of f(x) at x = 3.75 from the table

x	2.5	3.0	3.5	4.0	4.5	5.0
f(x)	24.145	22.045	20.225	18.644	17.262	16.047

15. Using Sterling's formula, find $\tan 16^{\circ}$ from the following data

x	0	5	10	15	20	25	30
tanx	0	0.0875	0.1763	0.2639	0.3640	0.4663	0.5774

16. Using Bessel's formula, find y(62.5) from the following data

x	60	61	62	63	64	65
y	7782	7853	7924	7993	8062	8129

17. Using Sterling's formula, find e^{-x} from the following data

x	1.72	1.73	1.74	1.75	1.76	1.77	1.78
e^{-x}	0.1791	0.1773	0.1775	0.1738	0.1720	0.1703	0.1686

18. Using Sterling's formula and Bessel's formula, find y_{35} given that $y_{20} = 512$, $y_{30} = 439$, $y_{40} = 346$, $y_{50} = 243$.
19. Using Gauss backward formula, find the value of $\sin 45^{\circ}$ from the following data

x	20	30	40	50	60	70
sinx	0.34202	0.502	0.64279	0.76604	0.86603	0.93969

Unit – III Interpolation for unequal intervals

2 Marks

1. Define Divided difference formula.
2. Write a formula for n^{th} divided difference formula.
3. Form the divided difference table for the following data:

x	5	15	22
y	7	36	160

4. What advantages had Lagrange's formula over Newton's formula?

5. State Newton's divided difference formula.
6. Find out the divided difference of y_x, given that

x	1	2	4	7	12
y	22	30	82	106	206

7. Find the second degree polynomial from the following data

x	1	2	4
y	4	5	13

8. State Lagrange interpolation formula.
9. If f(x) and g(x) are two functions are α, β are constant, then $\Delta[\alpha f(x) + \beta g(x)] = \alpha\Delta f(x) + \beta\,\Delta g(x)$. Show that Δ is a linear.

5 Marks

1. Find out the divided difference table of y_x, given that

x	1	2	4	7	12
y_x	22	30	82	106	206

2. Given the data

x	0	1	2	5
f(x)	2	3	12	147

Find the cubic function of x.

3. Show that the n^{th} divided difference of a polynomial of the n^{th} degree are constant.
4. Show that $\underset{bcd}{\Delta^3}\,(1/a) = \frac{-1}{abcd}$.
5. The following table gives the normal weights of badies during the first few months of life.

Age in months:	2	5	8	10	12
Weight in kgs:	4.4	6.2	6.7	7.5	8.7

Estimate by Lagrange's method, the normal weight of a 7 month old bady.

6. Find the third divided difference with arguments 2,4,9,10 of the function $f(x) = x^3-2x$.
7. Derive Newton's divided difference formula for an unequal interval.
8. Given $y_0 = -12$, $y_1 = 0$, $y_3 = 6$, $y_4 = 12$. Find y_2 using Newton's divided difference formula.
9. If $f(x) = 1/x^2$, find the divided difference f(a,b,c).
10. Find the equation y = f(x) of least degree and passing through the points (-1,-21), (1,15), (2,12), (3,3). Find also y at x=0.
11. Find 1955 & 1965.

Year	1940	1945	1950	1955	1960	1965	1970
Population	20	22	26	-	35	-	43

12. Derive Lagrange's interpolation formula.

13. Using Lagrange's interpolation formula find the value corresponding to x=10 from the following data

x	5	6	9	11
y	12	13	14	16

14. Interpolate the value of y at x=5 using Lagrange interpolation formula for the following data

x	1	2	3	4	7
y	2	4	8	16	128

Find the value of f(6).

15. If y_1=-4, y_3=12, y_4 = 19 and y_x = 7. Find x.
16. Prove that Lagrange formula can be put in the form

$$P_n(x) = \sum_{r=0}^{n} \frac{\varphi(x)\, f(x_r)}{(x - x_r)\varphi'(x_r)} \quad where\ \varphi(x) = \prod_{r=0}^{n}(x - x_r).$$

10 Marks

1. Using Newton's divided difference formula, fine the value of f(8) from the given data:

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

2. Given the values

x	14	17	31	35
f(x)	68.7	64.0	44.0	39.1

Find the value of f(27).

3. Obtain the best estimates for the missing figures below.

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
y	0.135	?	0.111	0.100	?	0.082	0.074

4. Find the polynomial of fifth degree from the following data u_0 = -18, u_1 = 0, u_3 = 0, u_5 = -248, u_6 = 0, u_9 = 13104.
5. Find the values corresponding to y = 12 using lagranges inverse formula from the following table.

x	1.2	2.1	2.8	4.1	4.9	6.2
y	4.2	6.8	9.8	13.4	15.5	19.6

Unit - IV Inverse Interpolation

2 Marks

1. Define Inverse interpolation.
2. Give the inverse Lagrange's interpolation formula.
3. State Montmort's theorem.
4. Find the value of $\Delta^{-1}x^{(n)}, n \neq 1$.
5. Sum to n terms of the series whose x^{th} term is $\frac{1}{(x+1)(x+2)(x+3)}$.
6. What are the restrictions have to be imposed in Inverse interpolation?
7. Give the Iteration method formula.
8. Write about Reversion of series method.
9. Sum to n terms 1.2.3+2.3.4+3.4.5+……..
10. Write Newton's second approximation formula to find x by iterative method.
11. Write aninterpolation formula for Reversion of series method using Newton's formula.
12. Write an interpolation formula for Reversion of series method using Stirling's formula.
13. Write an interpolation formula for Reversion of series method using Bessel's formula.

5 Marks

1. Obtain the value of x when f(x) = 19, by Lagrange's method, given that for x=0,1,2, f(x) = 0,1,20 respectively.
2. Find the sum to n terms $1.3^2 + 3.5^2 + 5.7^2 +$……….
3. Sum the series to n terms 1.2.3 + 2.3.4 + 3.4.5 + ……..
4. Use Inverse Lagrange's formula to obtain the root of the equation f(x)=0, given that f(30) = -30, f(34) = -13, f(38) = 3 and f(42) = 18.
5. Find the sum to n terms of the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots \ldots \ldots$
6. From the following data, find the value of x when y = 13.5 using Lagrange's formula.
7. From the following value of y = f(x) are given

x	10	15	20
f(x)	1754	2648	3564

   Find the value of x for f(x) = 3000 by successive approximation method.
8. Solve x = ½ + sinx by Iterative method.
9. Write a derivation for Reversion of series method.
10. Explain about Iteration method.
11. Find the root of $x^3 - 6x - 11 = 0$ lies between 3 & 4.
12. Sum to n terms of the series 2.5+ 5.8 + 8.11 + 11.14 + ……..
13. Find the sum of $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$.

14. Sum to n terms $\frac{2}{2.5.8}+\frac{5}{5.8.11}+\frac{7}{8.11.14}+\cdots\ldots\ldots..$
15. Using Lagrange's inverse find x to one decimal place for f(x) = 14,

x	0	5	10	15
f(x)	16.35	14.88	13.59	12.46

10 Marks

1. Sum the series 1.3 + 3.5x+ 5.7x^2 + 7.9x^3 + ………..
2. Sum the series to 2,12,36,80,150,252,……
3. Sum to n terms the series whose x^{th} term is $\frac{x+3}{x(x+1)(x+2)}$.
4. Using Iteration method find x when y = 5

x	1.8	2.0	2.2	2.4	2.6
y	2.9	3.6	4.4	5.5	6.7

5. Table y = x^3 for x = 2,3,4,5 & Calculate the cube root of 10 correct to three decimal places.
6. The following table gives the values of sinhx for certain equidistant values of x. If sinhx = 62. Find x.

x	4.80	4.81	4.82	4.83	4.84
Sin hx	60.7511	61.3617	61.9785	62.6015	63.2307

Unit – V

2 Marks

1. State the condition of convergence of Gauss – Seidal method.
2. Write any two direct method to solve simultaneous linear equations.
3. Compare Gauss elimination and Gauss Seidal methods.
4. Using Gauss-Seidal method find only first iteration 8x-y+z = 18 , 2x +5y-2z = 3, x+y-3z = -6.
5. Explain the method of complete pivoting.
6. Solve by Gauss Jordan method x+y = 2, 2x+3y = 5.
7. Define Diagonally dominant matrix with example.

5 Marks

1. Solve the system by Gauss elimination method 2x+3y-z = 5, 4x+4y-3z = 3 and 2x-3y+2z = 2 .
2. Solve the system using Gauss Jordan method 10x+y+z = 12, 2x+10y + z = 13, x+y+5z = 7.

3. Solve the system by Gauss elimination method x+2y+z = 3, 2x+3y+3z = 10 and 3x-y+2z = 13 .
4. Solve the system by Gauss Seidal method 27x+6y-z = 85, 6x+15y+2z = 72 and x+y+54z = 110 .
5. Solve the system by Gauss Jordan method 2x+y+z = 10, 3x+2y+3z = 18 and x+4y+9z = 16 .
6. Solve the system by Matrix inversion method x+2y+3z = 10, 2x-3y+z = 1 and 3x+y-2z = 9 .

10 Marks

1. Solve the system by Gauss Seidal method 28x+4y-z = 32, x+3y+10z = 24 and 2x+17y+4z = 35 .
2. Find the Gaussian elimination, the inverse of $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$.
3. Using Gauss-Seidal method, solve the following system of equations, tabulating the results upto the 5^{th} iteration 8x-3y+2z = 20, 4x+11y-z = 33 and 6x+3y+12z = 35 .
4. Solve the system by Crout's method x+y+z = 12, 2x+10y+z = 13 and 2x+2y+10z = 14 .

MATHEMATICS FOR COMPETITIVE EXAMINATIONS II – CSMA 32

UNIT :1 CHAIN RULE & TIME AND WORK

1.If 8 men can reap 80 hectares in 24 days ,Then how many hectares can 36 men reap in 30 days?

2.If 7 spiders makes 7 webs in 7 days. Then 1 spider will make 1 web in how many days?

3.Some persons can do a piece of work in 12 days. Two times the number of such persons will do half of that work?

4.If 15 men, working 9 hours a day , can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

5.If 15 toys cost Rs.234,What do 35 toys cost?

6.If the wages of 6 men for 15 days be Rs.2100,Then find the wages of 9 men for 12 days?

7.The price of 357 mangoes is Rs. 1517.25. What will be the approximate price of 9 dozens of such mangoes?

8.A can finish a work in 24 days ,B in 9 days and C in 12 days. B and C starts the work .But are forced to leave after 3 days. The remaining work was done by A ?

9.4men and 6 women can complete a work in 8 days, While 3 men and 7 women can complete it in 10 days .In how many days will 10 women complete it ?

10.A is twice as good as workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

11.A and B can do a work in 12 days, B and C in 15 days, C and A in 20 days .If A,B and C work together , they will complete the work?

12.A man can do a piece of work in 5 days ,but with the help of his son, he can do it in 3 days. In what time can the son do it alone?

13.A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work?

14.A sum of money is sufficient to pay A’s wages for 21 days and B’s wages for 28 days .The same money is sufficient to pay the wages of both?

15.10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?

16.Which of the following trains is the3 fastest?

17. A person crosses a 600 m long street in 5 minutes .What is his speed in km per hour?

18.A is twice as fast as B and B is thrice as fast as C is the journey covered by C in 54 minutes will be covered by B?

19.3 pumps, working 8 hours a day, can empty a tank in 2 days. How many hours a day must 4 pumps work to empty the tank in 1 day?

20. If the cost of *x* metres of wire is d rupees, then what is the cost of *y* metres of wire at the same rate?

21. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 minutes?

22. A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is

23. 39 persons can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day, complete the work?

24. A man completes $\frac{5}{8}$ of a job in 10 days. At this rate, how many more days will it takes him to finish the job?

25. If a quarter kg of potato costs 60 paise, how many paise will 200 gm cost?

26. In a dairy farm, 40 cows eat 40 bags of husk in 40 days. In how many days one cow will eat one bag of husk?

27. A wheel that has 6 cogs is meshed with a larger wheel of 14 cogs. When the smaller wheel has made 21 revolutions, then the number of revolutions mad by the larger wheel is:

28. If 7 spiders make 7 webs in 7 days, then 1 spider will make 1 web in how many days?

29. A flagstaff 17.5 m high casts a shadow of length 40.25 m. The height of the building, which casts a shadow of length 28.75 m under similar conditions will be:

30. In a camp, there is a meal for 120 men or 200 children. If 150 children have taken the meal, how many men will be catered to with remaining meal?

31. An industrial loom weaves 0.128 metres of cloth every second. Approximately, how many seconds will it take for the loom to weave 25 metres of cloth?

32. 4 mat-weavers can weave 4 mats in 4 days. At the same rate, how many mats would be woven by 8 mat-weavers in 8 days?

33. A can lay railway track between two given stations in 16 days and B can do the same job in 12 days. With help of C, they did the job in 4 days only. Then, C alone can do the job in:

34.Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio between the capacity of a man and a woman?

UNIT:2 TIME AND DISTANCE

1.A car is running at a speed of 108 kmph .What distance will it cover in 15 seconds?

2.Three persons are walking from a place A to another place B .Their speeds are in the ratio of 4:3:5.The time ratio to reach B by these persons will be?

3.A walks at 4 kmph and 4 hours after his starts, B cycles afte4r him at 10 kmph. How far from the start does B catch up with A?

4.In coveting a distance of 30 km, Abhay takes 2 hours more than sammeer.If Abhay doubles his speed , Then he would take 1 hour less than Sameer . Abhay’s speed?

5.Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph .For how many minutes does the bus stop per hour?

6. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minutes?

7. Two trains A and B start simultaneously in the opposite direction from two points P and Q and arrive at their destinations 16 and 9 hours respectively after their meeting each other. At what speed does the second train B travel if the first train travels at 120 km/h?

8. Two horses start trotting towards each other, one from A to B and another from B to A. They cross each other after one hour and the first horse reaches B, 5/6 hour before the second horse reaches A. If the distance between A and B is 50 km. what is the speed of the slower horse?

9. The speed of a car increases by 2 kms after every one hour. If the distance travelling in the first one hour was 35 kms. what was the total distance travelled in 12 hours?

10. A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. The average speed for the first 320 km of the tour ?

11. The distance from town A to town B is five miles. C is six miles from B. Which of the following could be the maximum distance from A to C?

12. A person goes to his office at 1/3rd of the speed at which he returns from his office. If the average speed during the whole trip is 12 m /h .what is the speed of the person while he was going to his office?

13. A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?

14. An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in 1 $\frac{2}{3}$ hours, it must travel at a speed of:

15. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is:

16. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is:

17. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

20. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is:

21. A man complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km/hr and second half at the rate of 24 km/hr. Find the total journey in km.
22. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 4 hours, then the speed of the first train is:
23. A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. The average speed for the first 320 km of the tour is:
24. A car travelling with $\frac{5}{7}$ of its actual speed covers 42 km in 1 hr 40 min 48 sec. Find the actual speed of the car.
25.It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the cars is:
26. Robert is travelling on his cycle and has calculated to reach point A at 2 P.M. if he travels at 10 kmph, he will reach there at 12 noon if he travels at 15 kmph. At what speed must he travel to reach A at 1 P.M.?
27. A farmer travelled a distance of 61 km in 9 hours. He travelled partly on foot @ 4 km/hr and partly on bicycle @ 9 km/hr. The distance travelled on foot is:

UNIT:3 PROBLEMS ON TRAIN

1.A train speeds past a pole in 15 seconds and a platform 100m long 25 seconds .Find its length?

2.A speed of 14 metres per second is the same as?

3.The length of the bridge ,Which a train 130 m long and travelling at 45 kmph can cross in 30 seconds ?

4.A train moves with a speed of 108 kmph. Its speed in metres per second?

5.A train 132 m long passes a telegraph pole in 6 seconds .Find the speed of the train ?

6. A man sitting in a train which is running at a speed of 100 km/hr saw a goods train which is running in opposite direction towards him. The goods train crosses the man in 8 seconds. If the length of goods train is 300 meters, find its speed.

7. Two trains of equal length are moving in same direction on parallel tracks at speed of 92 km/hr and 72 km/hr respectively. The faster train crosses the slower train in 18 seconds. Find the length of each train.

8. Two trains of length 140 meters and 166 meters are moving towards each other on parallel tracks at a speed of 50 km/hr and 60 km/hr respectively.In what time the trains will cross each other from the moment they meet?

9. Two trains running in opposite direction cross a man standing on the platform in 36 seconds and 26 seconds respectively. The trains cross each other in 30 seconds. What is the ratio of their speeds?

10. Two trains of length 120 meters and 140 meters are moving in the same direction on parallel tracks at speed of 82 km/hr and 64 km/hr. In what time the first train will cross the second train?

11. A train of length 200 meters takes 12 seconds to cross a man who is running at a speed of 10 km/hr in opposite direction of the train. What is the speed of the train?

12. Two trains are moving towards each other with speeds 40 km/hr and 45 km/hr from different stations P and Q. When they meet the second train from station Q has covered 20 km more distance than the first train which starts from station P. What is the distance between the two stations?

13. Two trains of length 125 meters and 115 meters are running on parallel tracks. When they run in the same direction the faster train crosses the slower train in 30 seconds and when they run in opposite direction they cross each other in 10 seconds. What is the speed of each train?

14. A train crosses two men who are running in the direction of train at 4 km/hr and 8 km/hr in 18 and 20 seconds respectively. Find the length of train.

15. A train moving at 108 km/hr crosses a platform in 30 seconds. Then it crosses a man running at 12 km/hr in the same direction of train in 9 seconds. What is the length of train and platform?

16. Two, trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is:

17. Two trains are running at 40 km/hr and 20 km/hr respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train?

18. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds respectively. The length of the train is:

19. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/hr. The other one walks at 5.4 km/hr. The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train?

20. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m. and travels towards B at 20 kmph. Another train starts from B at 8 a.m. and travels towards A at a speed of 25 kmph. At what time will they meet?

21. How many seconds will a 500 metre long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr?

22. Two trains are running in opposite directions with the same speed. If the length of each train is 120 metres and they cross each other in 12 seconds, then the speed of each train (in km/hr) is:

23. A train moves past a telegraph post and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train?

24. A 270 metres long train running at the speed of 120 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 seconds. What is the length of the other train?

25. Two trains of equal length are running on parallel lines in the same direction at 46 km/hr and 36 km/hr. The faster train passes the slower train in 36 seconds. The length of each train is:

26. A jogger running at 9 kmph alongside a railway track in 240 metres ahead of the engine of a 120 metres long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?

UNIT:4 BOATS AND STREAMS

1.A man can row upstream at 8 kmph and downstream at 13 kmph.

The speed of the stream?

2.A man can row upstream at 7kmph and downstream at 10 kmph. Find man rate in still water and the rate of current?

3.The speed of a boat in still water in 10 km/hr .If it can travel 26 km downstream and 14km upstream in the same time .What is the speed of the stream?

4.If a man rows at the rate of 5 kmph in still water and his rate against the current is 3.5 kmph. Then the mans rate along the current?

5.A man can row 18 kmph in still water .It takes him thrice as long to row up as to row down the river. Find the rate of stream?

6. A man swims 12 km downstream and 10 km upstream. If he takes 2 hours each time, what is the speed of the stream?

7. A boat covers 800 meters in 600 seconds against the stream and returns downstream in 5 minutes. What is the speed of the boat in still water?

8. A man can row a boat at a speed of 20 km/hr in still water. If the speed of the stream is 5 km/hr, in what time he can row a distance of 75 km downstream?

9. The speed of a boat in still water is 5km/hr. If the speed of the boat against the stream is 3 km/hr, what is the speed of the stream?

10. A man swimming in a river which is flowing at $3\frac{1}{2}$ km/hr finds that in a given time he can swim twice as far downstream as he can swim upstream. What will be his speed in still water?

11. A boat takes 6 hours to move downstream from point P to Q and to return to point P moving upstream. If the speed of the stream is 4 km/hr and speed of the boat in still water is 6 km/hr, what is the distance between point P and Q?

12. A motorboat travels 16 km in 2 hours against the flow of river and travels next 8 km along the flow of the river in 20 minutes. How long will it take motorboat to travel 48 km in still water?

13. A boat covers 6 km upstream and returns back to the starting point in 2 hours. If the flow of the stream is 4 km/hr, what is the speed of the boat in still water?

14. A man can row 9[1/3] km/hr in still water. He finds that it takes thrice as much time to row upstream as to row downstream (same distance). Find the speed of the current.

15. The velocity of a boat in still water is 9 km/hr, and the speed of the stream is 2.5 km/hr. How much time will the boat take to go 9.1 km against the stream?

16. A boatman goes 2 km against the current of the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will it take to go 5 km in stationary water?

17. A man can row three-quarters of a kilometre against the stream in 11 $\frac{1}{4}$ minutes and down the stream in 7 $\frac{1}{2}$ minutes. The speed (in km/hr) of the man in still water is:

18. Speed of a boat in standing water is 9 kmph and the speed of the stream is 1.5 kmph. A man rows to a place at a distance of 105 km and comes back to the starting point. The total time taken by him is:

19. A man rows to a place 48 km distant and come back in 14 hours. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. The rate of the stream is:

20. A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water?

21. The speed of a boat in still water in 15 km/hr and the rate of current is 3 km/hr. The distance travelled downstream in 12 minutes is:

22. A boat takes 90 minutes less to travel 36 miles downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 mph, the speed of the stream is:

23. A man can row at 5 kmph in still water. If the velocity of current is 1 kmph and it takes him 1 hour to row to a place and come back, how far is the place?

24. A boat covers a certain distance downstream in 1 hour, while it comes back in 1 $\frac{1}{2}$ hours. If the speed of the stream be 3 kmph, what is the speed of the boat in still water?

25. A motorboat, whose speed in 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is:

26. In one hour, a boat goes 11 km/hr along the stream and 5 km/hr against the stream. The speed of the boat in still water (in km/hr) is:

27. A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?

28. A man's speed with the current is 15 km/hr and the speed of the current is 2.5 km/hr. The man's speed against the current is:

29. A boat can travel with a speed of 13 km/hr in still water. If the speed of the stream is 4 km/hr, find the time taken by the boat to go 68 km downstream.

UNIT :5 ALLIGATION AND MIXTURE

1.In what ratio must be mixed with milk to gain 20% by selling .Find mixture at cost price?

2.In what ratio must rice at Rs.9.30 per kg be mixed with rice at Rs.10.80 per kg .What is the mixture be worth Rs.10 per kg?

3.How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs.20.Find the mixture worth Rs.$10\frac{2}{3}$litre?

4.A dishonest milkman professes to sell his milk at cost price .But he mixes it with water and there by gains 25% . Find the percentage of water in the mixture?

5.In what ratio must water be mixed with milk costing Rs.12 per litre. To obtain a mixture worth of Rs.8 per litre?

6. A 60 liter mixture of milk and water contains 10% water. How much water must be added to make water 20% in the mixture?

7.700 ml of a mixture contains water and milk in the ratio 2:8. How much water must be added to the mixture so that the ratio of water and milk becomes 3:8?

8. A rice dealer bought 60 kg of rice worth Rs. 30 per kg and 40 kg of rice worth Rs. 35 per kg. He mixes the two and sells the mixture at Rs. 40 per kg. What is the percentage profit in this deal?

9. 1/2 and 1/4 parts of two bottles are filled with milk. The bottles are then filled completely with water and the content of bottles is poured into a container. Find the ratio of the milk and water in the container?

10. A bottle of whisky contains 40% alcohol. If we replace a part of this whisky by another whisky containing 20% alcohol, the percentage of alcohol becomes 28%. What quantity of whisky is replaced?

11. An alloy has copper and zinc in the ratio of 6:3 and another alloy has copper and tin in the ratio of 8:6. The equal weights of both the alloys are melted to form a new alloy. What will be the weight of tin per kg of the new alloy?

12. A shopkeeper mixes 60 kg of sugar worth Rs. 30 per kg with 90 kg of sugar worth Rs. 40 per kg. At what rate he must sell the mixture to gain 20%?

13. A 20 liter mixture contains 30% alcohol and 70% water. If 5 liters of water is added to the mixture, what will be the percentage of alcohol in the new mixture ?

14.Tea worth of Rs. 135/kg & Rs. 126/kg are mixed with a third variety in the ratio 1: 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be____?

15. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine the cask hold originally?

16. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

17. A can contains a mixture of two liquids A and B is the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

18. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?

19. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 kg?

20. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:

21. How many kilogram of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?

22. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

23. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is:

24. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg.

25. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10%?

26. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is:

27. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of water is 16 : 65. How much wine did the cask hold originally?

28. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is:

NME - General Commercial Knowledge – CNCM 37

SECTION-A(2MARKS)

1. What is business?
2. What is profession?
3. What is industry?
4. Define small scale industry?
5. What is large scale industry?
6. What is Foreign Trade?
7. What is Export?
8. What is Entrepot Trade?
9. What is meant by employment?
10. Define Import trade
11. What is Genetic industry?
12. What is Commerce?
13. 13 Define the term Business
14. What do mean by the term ethics?
15. Define the term business ethics.
16. Define social responsibility?
17. State the factors that influence business ethics?
18. Mention a few unethical practices in business?
19. Mention a few of the responsibilities of business towards customers?
20. Mention a few of the responsibilities of business towards shareholder?
21. Define Business enterprise
22. What is Private sector enterprise?
23. What is Joint Stock Company?
24. Who is sole trader?
25. What do you mean by partnership deed?
26. What is a partner by estoppels?
27. Who is a minor?
28. What is sleeping partner?
29. What are the different types of company meetings?
30. State the features of company meetings?
31. What are the different kinds of company meetings of shareholders?
32. What is the first meeting held by a company?
33. What is meant by the term agenda?
34. What is a resolution?
35. State the types of resolution?
36. What is a company?
37. What is a promotion?

38. How is promoter given remuneration?
39. What is the legal position of a promoter?
40. What are the steps in promotion?
41. What is a private limited company?
42. What is memorandum of association?
43. Whatare articles of association?
44. What is prospectus?
45. What are the types of shares?
46. Define co-operative enterprise.
47. What are the objectives of co-operative?
48. Who are the pioneers of the co-operative?
49. Mention any five features of co-operative?
50. What are the types of co-operative enterprises?
51. What is the voting principle in co-operative?
52. Write the short note on consumer co-operative?
53. What is the objective of forming housing co-operatives?
54. State the requirements for constituting co-operative?
55. State the origin of public enterprises?
56. What are the forms by which public enterprises are organized?
57. What is meant by departmental undertaking?
58. Write a short note on statutory corporations?
59. Briefly explains the term Government Company?
60. Mention a few examples of departmental undertakings?
61. Mention a few examples of Government Company?
62. Mention a few examples of statutory corporations?
63. Define the term plant location.
64. What are the factors of the Weber's theory?
65. What is meant by split location?
66. What are the factors influencing plant location?
67. Mention a few factors in selection of site?
68. State the agglomerative factors?
69. State the deglomerative factors?
70. What do you mean by plant layout?
71. Define the term plant layout.
72. Mention the two objectives of plant layout?
73. State the characteristics of a good layout.
74. What are the advantages of good layout workers?
75. Point out the factors influencing of good layout?
76. What are the types of plant layout?
77. What is meant by product layout?

78. Give a outline of process layout?
79. What is meant by combined layout?
80. Difference between product layout and process layout?
81. 80. What is meant by the term size of firm? .‘
82. Distinguish between firm and industry?
83. Mention the various criteria to measure the size of firm?
84. What is meant by a representative firm?
85. Give a brief outline of equilibrium?
86. What are the objectives of DICs?
87. What are the resources available to DICs?
88. Givea brief outline of the structure to DICs?
89. Mention the activities ofDICs?
90. Define a industrial estate
91. What is a stock exchange?
92. Define a stock exchange
93. Who are sub brokers?
94. What is a depository?
95. Define a business combination.
96. What are the objectives ofbusiness combination?
97. What is a trade association?
98. Define at the term trade association
99. What are chambers of commerce?
100. Mention the benefits of chambers of commerce?

SECTION-B(5MARKS)

1. Define Business. State its characteristics?
2. Explain the objectives of Business?
3. How do you classifybusiness activities?
4. What are the aids to trade?
5. Distinguish between Trade and Commerce?
6. Distinguish between Business and profession?
7. State the features of sole trader?
8. Define Partner. State its features?
9. What are the rights of partner?
10. What are the duties of partner?
11. State the procedure for registration of a partner?
12. What is implied authority of a partner?
13. What is a statutory report? State its contents?
14. What is the legal position of a promoter?
15. Distinguish between memorandum and articles of association?

16. Explain the principles of co-operative societies?
17. What are co-operative credit societies?
18. Write the short notes on (a) consumer (b) producers co-operative?
19. State the features of departmental undertakings?
20. Give a brief outline of the characteristics of Government Company?
21. Mention the few problems of public enterprises?
22. Distinguish between StatutoryCorporation and Government Company?
23. Distinguish betweendepartmental undertakingsand Government Company?
24. What are the dynamics of industrial location?
25. Explain the advantages and disadvantages of stationary layout?
26. Explain the features of a goods plant layout?
27. What do you mean by the term Process layout? Explain its merits and demerit
28. What is meant by product layout? Explain its merits and demerits?
29. Discuss the various measures of size of firms.
30. Explain the various factors that influence the size of firms?
31. Give a detailed of the various types of firms?
32. What is an optimum firm? Explain the factors that influence optimum size?
33. Write short notes on ;(a) plant (b) firm (c)industry.
34. Explain the merits of large scale production?
35. State the motives of large scale production?
36. Explain the support of promotion of SSI's?
37. Discuss the performance of SSI's?
38. Give a brief account of SIDBI's activities?
39. State the activities of DIC's?
40. State the objectives and explain the features of industrial estates?
41. Explain the functions of Stock exchange?
42. Discuss the role, functions of SEBI?
43. Explain the causes of combinations?
44. Point out the disadvantages of combinations?
45. What meant by circular combinations? State out merits and demerits?
46. Explain a meaning, objectives of trade unions?
47. What are chambers of commerce? Mention some of their merits?
48. Distinguish between trade unions and chambers of commerce?
49. What are the similarities and differences trade and trade associations?
50. What are the types of combinations?

SECTION-C(10MARKS)

1. Explain the objectives of business?
2. What is commerce? State the importance of commerce?
3. What is Trade? State different types of trade?
4. Explain the factors that influence business ethics and its importance’s?
5. Discuss some of the unethical practices in various functional areas?
6. Explain the social responsibility and the need?
7. Explain the social responsibility of business stakeholders?
8. Explain the advantages and disadvantages of sole trader?
9. Explain the advantages and disadvantages of partnership deed?
10. What are contentof partnership deed?
11. Discuss various kinds of partners?
12. What are the duties and rights of partners?
13. What are the consequences of non registration of firms?
14. What are the advantages and disadvantagesregistration of firms?
15. Explain the provisions relating to the conduct of statutory meetings?
16. What isa resolution? Explain the different types of resolution?
17. Discuss the advantages and disadvantages of co operative societies?
18. Define co-operative. Explain their features and types?
19. State the objectives and achievements of public enterprises?
20. Explain in detail the problems faced by public enterprises?
21. Suggest measures to make public enterprises function effectively?
22. What are the advantages and disadvantages of departmental undertakings?
23. Explain the characteristics, merits and demerits of Government companies?
24. Discuss the features, advantages and disadvantages of statutory corporations?
25. Elucidate the factors that influence plant location?
26. What are theadvantages and disadvantages of locating a plant in cities and villages?
27. Discuss the various measures of size of firms?
28. Explain the various factors that influence the size of firms?
29. Give a detailed outline of the various types of firms?
30. Explain the factors that influence the optimum size of a firm?
31. Give a detailed outline of the economies of scale?
32. Explain in detail the various diseconomies scale?
33. Discuss the merits and demerits of small and large scale production?
34. Explain in detail the merits of SSI’s?
35. Examine a brief the problems faced by SSI’s?
36. Discuss the features, advantages of industrial estates?
37. Define a stock exchange. Mention its features and functions?
38. Give a detailed overview of the functioning of the OTCEI?
39. Discuss the role, functions and powers of SEBI?

40. Briefly explain in NSE?
41. Briefly explain in BSE?
42. Define the term combination. Elucidate the advantages and disadvantages ofcombinations?
43. Give a detailed explanation of the causes combinations?
44. Define a trade. Discuss its features, functions, and advantages?
45. What is a chamber of commerce? Enumerate their advantages?
46. What are the factors influence share prices?
47. Discuss in weakness of Indian markets?
48. Distinguish between partnership firm and Joint Stock Company?
49. Brief inJoint Stock Company?
50. Distinguish between public limited and private limited company?

III B.Sc., MATHEMATICS

SEMESTER V

ABSTRACT ALGEBRA – BMA 51

Unit-1

TWO MARKS

1. When a group is said to be abelian ?
2. Define group?
3. Give an example for subgroup?
4. If G is a finite group and a£G then prove that $a^{o(G)} = e$?
5. State fermats theorem ?
6. Define the symmetric group?
7. If G is a group then S.T $(a.b)^{-1} = b^{-1}.a^{-1} \forall a, b \in G$.
8. Define : centre of a group
9. Define : supgroup
10. S.T identity element of a group is unique.
11. Give an example of a subgroup of the group of set of all integers with operation addition.
12. If G is abelian group for all $a, b \in G$ prove that $(ab)^n = a^n b^n$.
13. P.T in a group , inverse of any element is unique.
14. If G is a group then S.T $(a^{-1})^{-1}$ =a
15. If a ,b ,c are positive integere , a isrelatively prime to b and divides bc , prove that divides c .
16. Verify whether the set of all integers is a group under multiplication .
17. S .T by an example that union of subgroups need not be a subgroup .
18. P.T the intersection of two subgroup of G is also a subgroup .
19. Define a order of an element of a group.
20. Give all subgroup of G={1,-1,i,-i} under multiplication.
21. Give an example of finite cyclic group.
22. S.T the cancellation laws hold in a group G .
23. If G is a finite group whose under is a prime number P, then G is cyclic group.
24. If G is a group of order 24 and $a^{2002} = a^n$ with 0<n<24. Find n.

25. If G is a group of even order P.T it has an element a different from the identity element e, such that $a^2 = e$.
26. How many groups are there for a group of order G ?
27. Write down all the subgroups of Z in the group of integers under addition ?
28. S.T if G is an abelian group iff $(a.b)^2 = a^2b^2 \forall\, a, b \in G$.
29. In a group $a^5 = e$,$aba^{-1} = b^2$ for $a, b \in G$ find o(b).
30. S.T N(a) is a subgroup of G.
31. What are all the subgroups of the multiplicative group of the 7^{th} roots of unity ?

5 Marks

32. If G is a group such that $(a.b)^2 = a^2.b^2$ for all $a, b \in G$ such that G must be abelian .
33. P.T every group of prime order is cyclic.
34. State and prove Euler's theorem.
35. If G is a group then P.T the identity element of G is unique and every $a \in G$ has a unique inverse in G.
36. If G is a finite group and $a \in G$ then P.T$o(a)/o(G)$.
37. P.T $(a.b)^n = a^n.b^n$. If G is an abelian group for all $a, b \in G$ and all integers n.
38. If G is a group in which $(a.b)^i = a^i.b^i$ for three consective integers i for all $a, b \in G$ S.T G is abelian.
39. If H is a non-empty finite subset of a group G and if the closure property is satisfied in H. S.T H is a subgroup of G.
40. S.T the centre of a group is a normal subgroup.
41. S.T the subgroup of a cyclic group is cyclic.
42. S.T a non-empty subset H of a group G is a subgroup of G iff $ab^{-1} \in G, \forall a, b \in H$.
43. If G is a group prove that
    (i) The identity element of G is unique.
    (ii) For all $a, b \in G$, $(a.b)^{-1} = b^{-1}.a^{-1}$.
44. P.T a non-empty subset H of a group is a subgroup of G iff
    (i) $a, b \in H$ implies that $ab \in H$.
    (ii) $a \in H$ implies that $a^{-1} \in H$.
45. Let H be a finite subset of a group G such that $ab \in H$ whenever $a \in H$ and $b \in H$. Then S.T H is a subgroup of G.
46. If $a \in H$, define $N(a) = \{x \in G / ax = xa\}$.S.T N(a) is a subgroup of G.

10 Marks

47. State and prove Lagrange's theorem.
48. Prove that HK is a subgroup of G iff HK=KH.
49. State and prove Lagrange's theorem.S.T its converse is not true.
50. (a) If G is a finite group and $a \in G$. P.T $o(a)/o(G)$.

(b) Let G be a group $\forall\, a, b, c \in G$. P.T $ab = ac \Rightarrow b = c$.

51. If S and T are two sets. S.T a mapping $\sigma: S \to T$ is a one-to-one correspondence between S and T iff there exist a mapping $\mu : T \to S$ such that $T \circ \mu$ and $\mu \circ \sigma$ are the identity mapping on S and T respectively.
52. If a and b are integers not both zero, then S.T the greatest common divisors (a,b) of a and b exists and that we can find integers m_0 and n_0 such that $(a, b) = m_0 a + n_0 b$.
53. If H and K are 2 subgroups of G. Define HK and give example to show that HK need not always be a subgroup .State and prove necessary and sufficient conditions for HK to be a subgroup of G.
54. Let H be a subgroup of G. S.T $a \equiv b \bmod H$ is an equivalence relation..
55. S.T the centre Z(G) of a group G and H is a subgroup of G.
56. Let G be a cyclic group and H, a subgroup of G. P.T H is a cyclic.
57. P.T any sub group of an infinite cyclic group is infinite.
58. If H is a non-empty finite subset of a group G and H is closed under multiplication. Then P.T H is a subgroup of G.

UNIT-2

2 Marks

1. Define cosets .
2. Define homomorphism .
3. If N is a normal subgroup of G then prove that $gNg^{-1} = N$ for every $g \in G$.
4. If G is a group of integers under addition and $\varphi : G \to G$ is defined $\varphi(x) = 2x \;\forall\, x \in G$, then S.T φ is a homomorphism .
5. Define kernel homomorphism .
6. If $\varphi : G \to \overline{G}$ is a homomorphism , then S.T $\varphi(e) = e$.
7. Define a normal subgroup of a group .
8. S.T every subgroup of an abelian group .
9. Let G be a group and $a \in G$ P .T $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$ is an isomorphism .
10. If h is a homomorphism of a group G_1 into G_2. S.T $h(e_1) = e_2$ where e_i is the identity of G_i.
11. If $\varphi : G \to \overline{G}$ is a homomorphism S.T $\varphi(x)^{-1} = \varphi(x^{-1})$ for all $\in G$.

12. Let $\varphi : G \to \overline{G}$ be a homomorphism of groups prove that a, $(e) = \overline{e}$, the unit element of $\overline{G}$.
13. Define quotient group .
14. If G is a group N is a normal subgroup of G then G/N is also a group is called quotient group .
15. Let G be the group of integers under addition and let G be for the integer$x \in G$, G=a defined by $\varphi : G \to \overline{G}$by $\varphi(x) = 2x$ then ø is a homomorphism .

5 Marks

16. P.T the normal of a homomorphism of a group is a normal subgroup.
17. If HK=KH then prove that HK is a subgroup of G.
18. If H and K are subgroup of G, S.T $H \cap K$ is a subgroup of G.
19. P.T for a group G and N is a normal subgroup of G iff $gNg^{-1} = N$ for every $g \in G$.
20. Let H and K be two subgroup of a group G, S.T HK is a subgroup of G iff HK=KH.
21. If G is a group and H is a subgroup of G of index 2 in G. P.T H is a normal subgroup of G.
22. P.T every quotient group of an abelian group is abelian.
23. S.T a subgroup N of a group G is a normal subgroup of G iff every left coset of N in G is also a right coset of N in G.
24. S.T kernel of a homomorphism is a normal subgroup.
25. Let N be a subgroup of G. S.T N is a normal subgroup of G iff $gNg^{-1} = N$ for every $g \in G$. Deduce that N is a normal subgroup of G iff every left coset of N in G is a right coset of N in G.
26. Let $\varphi : G \to \overline{G}$ be a homomorphism of groups. P.T

    (a) $\varphi(e) = \overline{e}$, the unit element of $\overline{G}$.

    (b) $\varphi(x^{-1}) = \varphi(x)^{-1}$ for all $x \in G$.

10 Marks

27. If K and H are finite subgroup of G , prove that $o(HK) = \frac{o(H)o(K)}{0(H \cap K)}$.
28. Let φ be a homomorphism of G onto $\overline{G}$ with kernel K, then prove that $\frac{G}{H} \simeq \overline{G}$.
29. Prove that HK is a subgroup of G iff HK=KH.
30. Let φ be a homomorphism of G_1 onto G_2 and let N_2 be a normal subgroup of G_2 and $N_1 = \{x \in G_1 : \varphi(x) \in N_2\}$ .Show that $\frac{G_1}{N_1} \simeq \frac{G_2}{N_2}$.
31. State and prove fundamental homomorphism theorem on groups.
32. Let $h : G_1 \to G_2$ be an onto homomorphism with kernel K, show that $\frac{G_1}{K} \simeq G_2$.
33. If φ is a homomorphism from a group G onto G' and if K is the kernel of φ, then S.T $\frac{G}{K} \simeq G'$.
34. If G is a group H a subgroup of G and S is the set of all right cosets of H in G, S.T there is a homomorphism θ is the largest normal subgroup of G which is contained in H.
35. If φ is a homomorphism of G into $\overline{G}$ with kernel K. P.T K is a normal subgroup of G.
36. Suppose that N and M are two normal subgroups of a group and that $N \cap M = (e)$. S.T for any $n \in N, m \in M, mn = nm$.
37. If φ is a homomorphism of a group G into a group $\overline{G}$ with kernel K, then prove that K is a normal subgroup of G. Also show that φ is one-to-one iff K=(e).
38. If G is a group and N is a normal subgroup of G, then S.T ${}^{G}/_{N}$ is also a group.
39. P.T every group is isomorphic to a subgroup of A(S) for some appropriate S.
40. Define a homomorphism between groups. When does it become an isomorphism? Give one example.
41. If N and M are two normal subgroups of G. P.T NM is also a normal subgroup of G.
42. Let φ be a homomorphism of G onto $\overline{G}$ with kernel K and $\overline{N}$ be a normal subgroup of $\overline{G}$. Let $N = \{x \in G : \varphi(x) \in \overline{N}\}$. S.T $\frac{\overline{G}}{N}$ is isomorphic to $\frac{G}{N}$ and $\frac{G}{N}$ is isomorphic to $\frac{(G/K)}{(N/K)}$.
43. S.T a homomorphism $f : R \to R'$ is 1-1 iff ker f $=\{0\}$.

44. Let $\theta : G \to G'$ be a homomorphism of groups . P.T ker θ is a normal subgroup of G . Further P.T θ is (1,1) mapping iff ker θ = {0}.
45. P.T subgroup H of G is normal in G iff xH=Hx for each $x \in G$.
46. Define a normal subgroup N of a group G. Define the quotient group $G/_N$. P.T $o(G/_N) = \frac{o(G)}{o(N)}$.
47. Let H be a normal subgroup of a group G. S.T the set $G/_H$ of left coste H in G from a group for the operation (aH)(bH) = abH.
48. P.T any infinite group is isomorphic to Z and any finite cyclic group of order n is isomorphic to Z_n.

Unit -III

2 Marks

1. If$\theta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$, $\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$ then find $\theta\varphi$.
2. Write the elements of S_3.
3. Define inner automorphism.
4. Define the even permutation.
5. What is meant by cycle of permutation ?
6. Express the permutation $\alpha = (1\ 2)(3\ 2\ 4)(5\ 6)(6\ 2\ 1)$ in S_6 as a product of disjoint cycles .
7. When do you say that a permutation is even ?
8. Define automorphism of a group and give an example .
9. Find the cycle of the permutation. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 & 9 \end{pmatrix}$
10. State Cayley's theorem.
11. Define a transposition.
12. Define a normal subgroup. Give a normal subgroup of the permutation group S_3.
13. Define for what m, an m-cycle is an even permutation.
14. Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ as a product of its cycles.
15. What is the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 5 & 1 & 3 \end{pmatrix}$?
16. Find the orbits and cycles of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$

17. Express $\binom{1\,2\,3\,4\,5\,6\,7\,8}{8\,7\,5\,4\,6\,2\,1\,3}$ as a product of its cycles.
18. Let G be a group and let $g \in G$. Prove that $\varphi : G \to G$ is defined by $\varphi(x) = gxg^{-1}$ is an automorphism of groups.
19. Which of the following permutations are a, even b, odd?
    (i) (1 2 3)(4 5 6)
    (ii) (1 2)(2 5 3 6)(1 3 2 4)
20. Write the permutation $\sigma = \binom{1\,2\,3\,4\,5\,6}{2\,1\,4\,5\,3\,6}$ as a product of 2-cycles.
21. S.T any finite group of order n is isomorphic to quotient group $Z/_N$. Where (Z , +) is a group of integers and M = (n).
22. S.T A_4 is the only subgroup of order 12 is S_4.
23. Express the permutation $\sigma = \binom{1\,2\,3\,4\,5\,6\,7\,8}{2\,3\,5\,8\,7\,6\,1\,4}$ a product of disjoint cycles .

5 Marks

24. If G is a group, then P.T A(G) the set of permutations of G is also a group.
25. P.T every permutation is product of 2-cycles.
26. S.T for any group G, Aut G is a group under composition of functions.
27. S.T the every permutation is the product of its cycles.
28. If G is a group S.T the set of all automorphism of G is also a group.
29. S.T the alternating group A_n is a normal subgroup of index 2 in S_n .
30. Determine which of the following are even permutation
    (i) $\binom{1\,2\,3\,4\,5\,6\,7\,8\,9}{2\,3\,4\,5\,1\,6\,7\,9\,8}$
    (ii) (1 2 3 4 5)(1 2 3)
31. S.T the collection of all automorphism of a group G is also a group.
32. S T alternating group is a normal subgroup of the symmetric group. **10 Marks**
33. State and prove Cayley's theorem.
34. State and prove Cayley's theorem on permutation.
35. Prove that (a) The identity permutation I is an even permutation.
    (b) The inverse of an even permutation is even.
    (c) The inverse of an odd permutation is odd

36. Write the cayley's composition table for S_3 and show that S_3 is a group which is not abelian.
37. Let G be a group and φ an automorphism of G.If $a \in G$ and o(a)>0. S.T $o\big(\varphi(a)\big) = o(a)$.
38. If S_n is the symmetric group of degree n and if A_n is the set of all even permutation in S_n, then P.T A_n is a normal subgroup of S_n and $o(A_n) = \frac{1}{2} < n$.
39. Every permutation is the product of its cucles.
40. If G is a group ,P.T the set of automorphism of G, is also a group.
41. S.T every element in S_n can be expressed as a product of disjoint cycles.
42. Define transposition an odd and even permutations.Express (1,2,3,4)(2,5,7,6)(5,7,8,9,10) as a product of transpositions.
43. S.T for $n \geq 3$,then subgroup generated by 3-cycles in A_n .

Unit – IV

2 Marks

1. If R is a ring and for all $a, b \in R$ then P.T a(-b)= (-a)b = - (ab) .
2. What is meant by ideal of R?
3. Define ring.
4. Define Quotient ring.
5. If U is an ideal of R and $1 \in U$ P.T U = R.
6. Define Zero divisors.
7. What is zero homomorphism ?
8. When do you say that a ring is of finite characteristic ?
9. Give an example of a subring of a non-commutative ring may be commutative.
10. Every right ideal need not be a left ideal – Give an example.
11. If R is a ring .S.T for $a \in R, a.0 = 0.a = 0$.
12. Define characteristic of an integral domain.
13. Define the kernel of a ring homomorphism.
14. S.T every field is an integral domain.

5 Marks

15. P.T a finite integral domain is a field.
16. P.T a field F has a no proper ideal.
17. If R is a commutative ring and $a \in R$. P.T $aR = \{ar/r \in R\}$ is a two sided ideal of R.
18. P.T ant field is an integral domain.
19. P.T the intersection of two left-ideals of R is a left-ideal of R.
20. S.T kernel of a ring homomorphism is an ideal.
21. S.T the intersection of two subrings is also a subring.
22. If f is a homomorphism of a ring R into a ring R', then P.T
    (i) f(0)=0.
    (ii) f(-a)=-f(a) for all $a \in R$.
23. If D is an integral domain of finite characteristic, P.T characteristic is a prime number.
24. Let $J(\sqrt{2})$ be the set of all real number's of the form $+n\sqrt{2}$, when m and n are integers. Show that $\varphi : J(\sqrt{2}) \to J(\sqrt{2})$ defined by $\varphi\left(m + n\sqrt{2}\right) = m - n\sqrt{2}$ is a ring homomorphism.
25. If U is an ideal of the ring R. P.T $R/_U$ is a ring.

10 Marks

26. If U is an ideal of R, P.T $R/_U$ is a ring and is a homomorphism image of R.
27. State and prove fundamental theorem of homomorphism for ring.
28. If R is a commutative ring with unit element and M is an ideal of R, then P.T M is a maximal ideal of R iff $R/_M$ is a field.
29. If D is an integral domain and D is of finite characteristic , P.T the characteristic of D is a prime number.
30. State and prove the necessary and sufficient condition for a non-empty subset S of a ring R to be a subring.
31. S.T a commutative ring R with unit element is a field iff its ideals are (o) and R.
32. Every integral domain can be imbedded in a field.

Unit- V

2 Marks

1. Define maximal ideal .
2. Define the Euclidean Ring.
3. P.T $(a,b)\sim(c,d) \Leftrightarrow ad = bc$ is a transitive.
4. Define principal ideal ring.
5. Define greatest common divisor of two elements in a ring.
6. P.T a Euclidean ring possesses a unit element.

5 Marks

7. If P is a prime P.T ‘P’ is a maximal ideal in Z.
8. If [a.b]=[a’, b’] and [c,d]=[c’, d’] then P.T [a,b][c,d]=[a’ , b’][c’ , d’].
9. If R is an Euclidean ring then prove that any two elements a and b in R have a greatest common divisors d.
10. P.T if K ia any field which contains D then K contains a subfield isomorphic to F.
11. If R be a Euclidean ring and for $a, b, c \in R$, $a/_{bc}$ but (a,b) = 1 then S.T $a/_{c}$.
12. Let R be a commutative ring with unit element whose only ideal are (0) and R. S.T R is a field.
13. Let R be an Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R. S.T $d(a) < d(ab)$.
14. S.T Euclidean ring is a principal ideal ring.
15. S.T the ring of polynomials over a field is Euclidean ring.
16. If R is a commutative ring with unit element. S.T M ia a maximal ideal of R iff $R/_{M}$ is a field.
17. Let A be an ideal of an Euclidean ring R. Show that exist $a_0 \in A$ such that every element of A is of the form xa_0 for some $x \in R$.

10 Marks

18. P.T M is a maximal ideal of R iff $R/_{M}$ is a field.
19. P.T every integral domain can be imbedded into a field.
20. P.T the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R iff a_0 is a prime element of R.
21. S.T every non-zero element of a Euclidean ring R is either a unit in R (or) can be uniquely written as a product of a finite number of prime elements of R.
22. State and prove unique factorization theorem for Euclidean rings.

REAL ANALYSIS-I – BMA 52

2 MARKS

UNIT I

1. Define : Real valued function.
2. Define : Maximum and Minimum function.
3. Define : Constant function.
4. Define : Universal set.
5. Define : Characteristic function.
6. Define : One to One function.
7. Define : Inverse function.
8. Define : Countable or denumerable.
9. Define : Limit of sequence.
10. Define : Cantor set.

UNIT II

1. Define: Divergent sequence.
2. Define: Bounded sequence.
3. Define: Cauchy sequence.
4. Define: Operation on convergent sequence.
5. Evaluate: $\lim_{n\to\infty} \sqrt{n(}\sqrt{n+1}\text{-}\sqrt{n})$.
6. Define: Monotone sequence.
7. Find the limit superior and limit inferior for $\{S_n\}=\{1,2,3,1,2,3,\ldots\ldots\}$.
8. Define :Limit superior sequence.
9. If the limit superior of the sequence $\{s_n\}_{n=1}^{\infty}$ is equal to M. iprove that the limit superior of any subsequence of $\{s_n\}$ is less than M.
10. Define: Limit inferior of a sequence.

UNIT III

1. Define: Partial sum of series.
2. Define: Alternating series.
3. Define: Conditional converges.
4. Define: Absolute converges.
5. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges.
6. When do you say that the series $\sum_{n=1}^{\infty} a_n$converges.

7. State: Ratio test.
8. State: Root test.
9. State: Comparison test for absolute convergence.
10. Test the convergence of $\sum \frac{1}{(logn)^n}$

UNIT IV

1. Define :The class l^2.
2. Define : Metric space.
3. Define : Limit in metric space.
4. Define: convergent sequence in Metric space.
5. Define: Cauchy sequence in Metric space.
6. Show that every Cauchy sequence in R_d is convergent.
7. Define : Non increasing sequence.
8. Define : Non decreasing sequence.
9. When do you say that a function f(x) approaches to L when x tends to a?
10. Define : Norm of a sequence.

UNIT V

1. Define: Open ball.
2. Defne : Open sets.
3. Define : Interior point.
4. Show that Finite intersection of open sets open.
5. Show that Infinite intersection of open sets need not be open.
6. Define : closed sets.
7. Infinite union of closed sets need not be closed: Justify.
8. Define : Limit point of a set.
9. Define : Closure of a set.
10. Give an example of a set which is neither open nor closed.

5 MARKS

UNIT I

1. P.T The set of rational numbers is countable.
2. P.T The set of rational numbers in [0,1] is countable.
3. P.T if A is any non empty sub set of R i.e bounded below then A has a greatest upper bound in R.

4. Find the limit of the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n=1$.
5. P.T all sub sequence of a convergent sequence of real number converges to the same limit.
6. P.T if the sequence of real number $\{s_n\}_{n=1}^{\infty}$ is converges to L then any subsequence of

$\{s_{n_k}\}_{n=1}^{\infty}$ is also converges.

7. P.T the sequence $\{s_n\}_{n=1}^{\infty}$ *has no* limit where $s_n=n$.
8. P.T $\lim_{n\to\infty}\{\frac{2n}{n+4n^{1/2}}\}=2$.
9. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of non negative numbers and if $\lim_{n\to\infty} s_n = L$ then $L\geq 0$.
10. P.T The set R of all real number is uncountable

.UNIT II

1. If a sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.
2. A non decreasing sequence which is bounded above is convergent.
3. P.T The sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
4. If a sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is bounded.
5. A non increasing sequence which is bounded below is convergent.
6. A non increasing sequence which is not bounded below diverges to $-\infty$.
7. P.T If $0<x<1$ then $\{x^n\}_{n=1}^{\infty}$ converges to 0.
8. P.T if $1<x<\infty$ *then* $\{x^n\}_{n=1}^{\infty}$ diverges to ∞.
9. P.T if $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequence of real number .If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ if then $\lim_{n\to\infty} s_n - t_n = L-M$.
10. P.T If $\{s_n\}_{n=1}^{\infty}$ is the sequence of real numbers. Then $\lim_{n\to\infty} \text{Inf } s_n \leq \lim_{n\to\infty} \sup s_n$.

UNIT III

1. State and prove the necessary condition that a series be convergent.
2. P.T if $\sum_{n=1}^{\infty} a_n$ is a series of non negative numbers with $s_n=a_1+a_2+\ldots\ldots.+a_n$ ($n\in I$) then
   (a) $\sum_{n=1}^{\infty} a_n$ Converges if the sequence is $\{s_n\}_{n=1}^{\infty}$ bounded.
   (b) $\sum_{n=1}^{\infty} a_n$ Diverges if $\{s_n\}_{n=1}^{\infty}$ is not bounded.

3. P.T if $0 < x < 1$ then $\sum_{n=1}^{\infty} x^n$ converges to $\frac{1}{1-x}$
4. P.T if $x\geq 1$ then $\sum_{n=1}^{\infty} x^n$ diverges.
5. P.T the series $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$ is divergent
6. P.T if $\sum_{n=1}^{\infty} a_n$ converges absolutely then $\sum_{n=1}^{\infty} a_n$ converges.
7. P.T if $\sum_{n=1}^{\infty} a_n$ converges absolutely then both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ converge
8. P.T if $\sum_{n=1}^{\infty} a_n$ converges conditionally then both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ diverge

9 S.T$\sum_{n=1}^{\infty} \frac{2n}{n^2-4n+7}$ diverges.

10 S.T $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ is absolutely convergent for all x

UNIT IV

1. State and prove Schwarz inequality.
2. State and prove Minkowske inequality.
3. If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n\, a_{2^n}$converges then $\sum_{n=1}^{\infty} a_n$ converges
4. If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n\, a_{2^n} div$verges then $\sum_{n=1}^{\infty} a_n$ diverges.
5. If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} a_n$ converges

then $\lim_{n\to\infty} na_n$ = 0.

6. P.T if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a}[f(x) + g(x)] = L + M$.
7. P.T if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a}[f(x) - g(x)] = L - M$.
8. P.T if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a}[f(x)g(x)] = LM\ and$ if M≠ 0
9. P.T if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a}[f(x)/g(x)] = L/M\ \ and$ if M≠ 0.
10. P.T Let f be a non decreasing function on the bounded open interval (a,b) if f is bounded above on (a,b) then $\lim_{x\to \bar{b}} f(x)$ exist.

UNIT V

1. Show that continuous function of a continuous function is continuous.
2. Show that arbitrary intersection of closed sets is closed.
3. Show that every open ball in a metric space is open.
4. Show that the real valued function f is continuous at a $\epsilon\ R$ if and only if, whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of numbers converging to a then the sequence $\{f(x_{n)}\}_{n=0}^{\infty}$ converges to f(a).
5. If f is any family of closed subsets of a metric space M, then prove that$\bigcap_{F\in f} F$ is also closed.
6. If f and g are realvalued functions if f is continuous at a and fg is continuous at f(a) then prove that $g^{\circ}f$ is continuous at a.
7. Show that every open set G⊂R$'$can be written as union of finite or countable number of open intervals which are mutually disjoint.
8. Prove that the composition of two continuous functions is continuous.
9. Prove that the intersection of any finite number of open sets is open.
10. Let G be an open subset of the metric space M. Then prove that G$'$=M - G.

10 MARKS

UNIT I

1. If $A_1, A_2, \ldots\ldots$ are countable sets then $\bigcup_{n=1}^{\infty} A_n$ is countable .
2. The set $[0,1]=\{x / 0 \leq x \leq 1\}$ is countable.

UNIT II

1. Any bounded sequence of real number has a convergent sub sequence.
2. State and prove Nested interval thorem.
3. If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$ is convergent.

UNIT III

1. State and prove Leibnitz theorem.
2. State and prove Comparison test.
3. State and prove Ratio test.
4. State and prove Root test.

UNIT IV

1. Let $< M, \rho >$be a metric space and let a be a point in M .Let f and g be real valued function whose domain are subsets of M

if$\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = N$ then

$$(i) \lim_{x \to a}[f(x) + g(x)] = L + N.$$

$$(ii) \lim_{x \to a}[f(x) - g(x)] = L - N .$$

$(iii) \lim_{x \to a}[f(x)g(x)] = LN \text{ } and$ ifN$\neq 0$

$$(iv) \lim_{x \to a}[f(x)/g(x)] = L/N \text{ } and \text{ if } N \neq 0.$$

2. If $\sum_{n=1}^{\infty} a_n$ converges absolutely to A ,then prove that any rearrangement $\sum_{n=1}^{\infty} b_n$of$\sum_{n=1}^{\infty} a_n$ also converges absolutely to A.

UNIT V

1. Show that f is continuous if and only if the inverse image of every open set is open.
2. State and prove a necessary and sufficient condition that a function on a metric space be continuous.
3. Let E be a subset of the metric space M .Show that the point $x \in M$ is a limit point of E if and only if every open ball B[x, r] about x contains atleast one point of E.

4. If E is any subset of a metric space M, then show that $\bar{E}$ is closed.

Complex Analysis I – BMA 53

Section A – 2 Marks

UNIT - I

1. S.T $e^{(2+3\pi i)} = -e^2$.
2. P.T $|\sin z|^2 = \sin^2 x + \sinh^2 y$.
3. Define complex no.
4. S.T $(3,1)(3,-1)(\frac{1}{5}, \frac{1}{10}) = (2,1)$.
5. Derive the polar form of complex no.
6. P.T $\arg(z_1.z_2) = \arg z_1 + \arg z_2$.
7. P.T $\arg(\frac{z1}{z2}) = \arg z_1 - \arg z_2$.
8. P.T $|z_1.z_2| = |z_1|.|z_2|$.
9. P.T $|\frac{z1}{z2}| = \frac{|z1|}{|z2|}$.
10. P.T $\arg(z) = -\arg(\overline{Z})$.
11. Find arg[i(x+iy)], if arg(x+iy) = ∞.

UNIT - II

1. Define fn. of a complex variable.
2. Define limit of a fn.
3. Define continuity of a fn.
4. Define analytic fn.
5. Define regular fn.
6. Define singular point.
7. Define neighbourhood of a point.
8. Define differentiability at a point.
9. Prove that an analytic fn. with constant real part is constant.
10. Prove that an analytic fn. with constant imaginary part is constant.
11. Define Jordan arc.
12. Verify C.R equations for the fn f(z) = Re z .
13. P.T the function $f(z) = |z|^2$ is differentiable only at z =0 .
14. P.T $f(z) = e^x(\cos y - i\sin y)$ is no where differentiable.
15. P.T $f(z) = \bar{z}$ is no where differentiable.
16. P.T f(z) =Re(z) is not analytic at any point.
17. For what value of z, w = f(z) fails to be analytic where $z = \log(\rho+i\phi)$ & $w = \rho(\cos\phi+i\sin\phi)$.
18. Find the value of z for which the fn. w fails to be analytic where $z = e^{-v}(\cos u + i\sin u)$.

19. Find the value of z for which the fn. w fails to be analytic where w = u + iv& z = sinhucosv + icoshusinv.
20. Verify C.R equations for the fn f(z) = Re z .
21. P.T the function f(z) = $|z|^2$is differentiable only at z =0.

UNIT – III

1. P.T u = 2x – x^2 + $3xy^2$ is Harmonic.
2. Define Harmonic fn.
3. Define conjugate Harmonic fn.
4. What angle of rotation is produced by the transformation w = $\frac{1}{z}$ at z = 1 ?
5. Define the transformation : Translation.
6. Define : Bilinear transformation.
7. Define : Transformation.

UNIT – IV

1. Define Isogonal mapping.
2. Define Conformal mapping.
3. Define the transformation : Rotation.
4. Let f(z) = u + iv be an analytic fn. in a region D. Then v is a Harmonic conjugate of u iff u is a Harmonic conjugate of –v.
5. Any two Harmonic conjugate of a given Harmonic fn. u in a region D differ by a real constant.
6. Define invariant point.
7. Define Cross ratio.
8. Find a fixed point and normal form of a bilinear transformation w = $\frac{z-1}{z+1}$.
9. What is the form of a bilinear transformation which has one fixed point $\propto$ &other fixed point is ∞.

UNIT – V

1. P.T $\int_c \frac{dz}{z-a} = 2\pi i.$
2. Define simple closed curve.
3. Define simple rectifiable arc.
4. Define contour integral.

5. Define simply connected region.
6. Define Entire fn.
7. State Cauchy – Goursat's theorem.
8. State Cauchy integral formula.
9. State Cauchy integral formula for higher order derivatives.
10. State Gauss mean value theorem.
11. Using Goursat's integral formula, P.T $\int_c \frac{zdz}{(a-z^2)(z+i)} = \frac{\pi}{5}$, where C is a circle |z| = 2.
12. Using Goursat's integral formula, Find $\int_c \frac{cosz\, dz}{z(z^2+8)}$.
13. Using Goursat's integral formula, Find $\int_c \frac{(3z2+7z+1)\, dz}{z+1}$.

Section B – 5 Marks

UNIT – I

1. S.T $| z_1 + z_2| \leq | z_1|+ |z_2|$.
2. S.T $| z_1 - z_2| \leq | z_1|+ |z_2|$.
3. S.T $| z_1 - z_2| \geq | z_1|- |z_2|$.
4. S.T $| z_1 + z_2| \geq | z_1|- |z_2|$.
5. S.T $| z1 + z2|^2 + | z1 - z2|^2 = 2\{ | z1|^2 + | z2 |^2\}$.
6. S.T $| a + \sqrt{a2 - b2} | + | a - \sqrt{a2 - b2} | = |a + b| + |a - b|$.
7. P.T $| Re\, z | + | Im\, z| \leq \sqrt{2}|z|$.
8. S.T Z is purely real iff $Z = \overline{Z}$ and Z is purely imaginary iff $Z = - \overline{Z}$.
9. To find the equation of a straight line joining two points z_1&z_2 in the complex plane.
10. Find the locus of the point z satisfying the condition $| z – 1 | \geq 2$.
11. Find the modulus & argument of $\frac{1+2i}{1-(1-i)^2}$.

UNIT – II

1. S.T $\frac{dw}{dz} = e^{-i\theta}\frac{\partial w}{\partial r} = -\frac{i}{r} e^{-i\theta}\frac{\partial w}{\partial \theta}$.
2. P.T the real & imaginary part of an analytic fn. satisfies the Laplace equation.
3. If w = f(z) = u + iv is an analytic fn in a domain D. Then the curves $u = C_1, v = C_2$ form an orthogonal family.
4. State & prove the composition of continuous fn is itself continuous.
5. S.T an analytic fn. with constant modulus is constant.
6. S.T an analytic fn. with constant argument is constant.
7. If ϕ &$\psi$are the functions of x & y satisfies the Laplace equation then show that s + it is analytic where $s = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}$&$t = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}$.

8. S.T $f(z) = xy + iy$ is continuous everywhere but not analytic.
9. P.T $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies the Laplace equation & also find the analytic fn. f(z).
10. If $u = (x - 1)^3 - 3xy^2 + 3y^2$. Find f(z) whose real part is u & also find imaginary part of f(z).
11. Find the nature of the fn.f(z) = $\frac{x2y5(x+iy)}{x4+y10}$, when $z \neq 0$,
    i. = 0 , when z = 0,in a region including origin.

UNIT – III

1. P.T the real & imaginary part of an analytic fn. are Harmonic fn.
2. Derive Milne – Thompson Method.
3. State & prove necessary condition for the mapping w = f(z) is to be conformal.
4. P.T every bilinear transformation is the resultant of bilinear transformation.
5. P.T the resultant of product of two bilinear transformation is a bilinear transformation.
6. S.T the Cross ratio remains invariant under a bilinear transformation.
7. Find a fixed point and normal form of a bilinear transformation $w = \frac{(2+i)z-2}{z+i}$.
8. Find a fixed point and normal form of a bilinear transformation $w = \frac{3iz+1}{z+i}$.

UNIT – IV

1. Find the bilinear transformation that maps the point $Z_1 = 0$, $Z_2 = -i$, $Z_3 = -1$ into $W_1 = i$, $W_2 = 1$, $W_3 = 0$.
2. Find the image of the strip $2 < x < 3$ under $w = \frac{1}{z}$.
3. If $\propto$& β are the two given points and k is the constant. Then the equation $\left|\frac{z-\propto}{z-\beta}\right| = k$ represents the circle where $k \neq 1$.
4. P.T the equation $\left|\frac{z-\propto}{z-\beta}\right| = k$ where $k \neq 1$represents the circle to which $\propto$& β are inverse points.
5. If w = u + iv. P.T the transformation$\frac{1+z}{1-z}$ transforms the unit circle $|z| < 1$ into the half plane $u > 0$.
6. S.T the transformation $w = \frac{2z+3}{z-4}$transforms the circle $x^2+y^2-4x = 0$ into a straight line.
7. The transformation $z = \frac{i-w}{i+w}$. S.T the half of the w – plane $v \geq 0$ corresponds to the circle $|z| \leq 1$ in the z – plane.

8. S.T under the transformation w = $\frac{z-i}{iz-1}$, the region Im(z) ≥ 0 is mapped on to the region|w| ≤ 1.

UNIT – V

1. Evaluate $\frac{e^{2z}dz}{(z+1)^4}$, where C is the circle | z | = 3 .
2. Evaluate $\frac{e^{iz}dz}{(z+1)^4}$, where C is the circle | z | = 3 .
3. Find the value of $\int_c f(z)dz$ where f(z) = (y – x – 3x^2)i and C is a line segment from z = 0, z = 1+i.
4. Find the value of $\int$z^{2}dz where C is a line segment from z = 0 to z = 2+i.
5. Find the value of $\int$z^{2}dz where C is a line segment from z = 0 to z = 1+i.
6. Evaluate $\int_c (x^2 + iy)dz$ along the closed curve y = x^2, y = x.
7. State & Prove Cauchy inequality.
8. Evaluate $\int_c \frac{\tan\frac{z}{2}}{(z-z_o)^2}$ dz.
9. P.T $\frac{1}{2\pi i}\int_c \frac{z^3 - 2}{(z-z_o)^3}dz$ = 3z$_o$. where C is any closed curve described in the positive sense and z$_o$ is inside C.
10. If C is a simple closed curve. S.T $\frac{1}{2\pi i}\int_c \frac{e^{az}}{z^{n+1}}dz = \frac{a^n}{n!}$.
11. S.T $\frac{1}{2\pi i}\int_c \frac{e^{zi}}{z^2+1}dz$ = sin t, where t>0 & C is the circle | z | = 3.
12. Evaluate $\int_c \frac{z^3 - 2z+1}{(z-i)^2}dz$, where C is a circle | z | = 2.

Section C – 10 Marks

UNIT – I

1. Find all values of z such that e^z = -2 .
2. Find the locus of the point z satisfying the conditionarg($\frac{z-1}{z+1}$) = μ.
3. Find the locus of the point z satisfying the condition $\left|\frac{z-i}{z+i}\right| \geq 2$.
4. Find the locus of the point z satisfying the conditionarg($\frac{z-1}{z+1}$) = $\frac{\pi}{3}$.
5. Explain stereographic projection.

UNIT - II

1. State & Prove necessary and sufficient condition for f^1(z$_o$) does not exist.
2. Derive C-R equation.

3. Derive CR equation in polar form.
4. State & Prove sufficient condition for f(z) to be analytic.
5. Suppose that f(z) = u(x,y) + iv(x,y) , $z_o = x_o + y_o$ and $w_o = u_o + v_o$then
6. $\lim_{z \to zo} f(z) = w_o$.
7. $\lim_{(x,y) \to (xo, yo)} u(x,y) = u_o$.
8. $\lim_{(x,y) \to (xo, yo)} v(x,y) = v_o$.
9. Suppose that $\lim_{z \to zo} f(z) = w_o$and $\lim_{z \to zo} F(z) = W_o$. Then
10. $\lim_{z \to zo} [f(z) \pm F(z)] = w_o \pm W_o$.
11. $\lim_{z \to zo} [f(z) \cdot F(z)] = w_o W_o$.
12. If $W_o \neq 0$,then $\lim_{z \to zo} \frac{f(z)}{F(z)} = \frac{wo}{Wo}$.
13. If z = x + iy then$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$
14. S.T $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \log |f'(z)| = 0$.If w = f(z) is a regular fn. of z such that $f'(z) = 0$.
15. If f(z) = u + iv is an analytic fn. of z then $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|\mathrm{Re}[f(z)]|^2 = 2|f'(z)|^2$.
16. Iff(z) = u + iv is an analytic fn. of z and ψ is a fn. of x & y with differential coefficient of first & second order then
17. $(\frac{\partial \psi}{\partial x})^2 + (\frac{\partial \psi}{\partial y})^2 = [(\frac{\partial \psi}{\partial u})^2 + (\frac{\partial \psi}{\partial v})^2] |f'(z)|^2$.
18. $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = (\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2})|f'(z)|^2$.
19. If u = $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. Find the corresponding analytic fn. f(z) = u + iv and also find the imaginary part of f(z).
20. S.T f(z) = $\frac{x3\,(1+i) - y3\,(1-i)}{x2+y2}$when $z \neq 0$,
    i. = 0 when z = 0,
    b. is not analytic at the origin even though CR equations are satisfied at the origin.
21. S.T the fn. f(z) = $\sqrt{|(xy)|}$ is not regular at the origin, although C-R equation are satisfied at the point.

UNIT – III

1. S.T the fn. $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is Harmonic. Find the conjugate Harmonic fn. v and express u + iv as an analytic fn. of z.

2. S.T the fn. u = e^x(xcosy – ysiny) is Harmonic. Find the conjugate Harmonic fn. v and express u + iv as an analytic fn. of z.
3. State & prove sufficient condition for the mapping w = f(z) is to be conformal.
4. The mapping w = f(z) is conformal then f'(z) = 0 if f(z) is a regular fn. of z.

UNIT – IV

1. i) In general there are two values of z for which w = z, but there is only one invariant point if $(a-d)^2 + 4bc = 0$.

ii) If there are distinct invariant point p and q, the transformation may be put in the form $\frac{w-p}{w-q} = k\frac{z-p}{z-q}$.

iii) If there is only one distinct invariant point p, the transformation may be put in the form $\frac{1}{w-p} = \frac{1}{z-p} + k$.

2. Determine the bilinear transformation that maps the point -1 ,0, 1 in z – plane into the points 0, i, 3i in the w – plane.
3. Determine the bilinear transformation that maps the point 0, -i, -1 in z – plane into the points i, 1, 0 in the w – plane.
4. If $\propto$& β are the inverse points of the circle then the equation of the circle can be written as $\left|\frac{z-\propto}{z-\beta}\right| = k$ where $k \neq 1$ and k is the real constant.
5. P.T every bilinear transformation maps into circle or straight line into circle or straight line in a bilinear transformation, a circle transforms into circle & inverse points transforms into inverse points.
6. Find the mobiustransformation whichtransform the half plane Im(z) ≥ 0 into circle |w| ≤ 1.
7. Find the mobiustransformation whichtransform the half plane Re(z) ≥ 0 into unit circle |w| ≤ 1.
8. Find the mobiustransformation whichtransform the unit circle | z | ≤ 1 into unit circle |w| ≤ 1.
9. S.T the general bilinear transformation of the circle | z | ≤ ρ maps into the circle |w| ≤ ρ′ is w = $\rho\rho' e^{i\lambda}\left[\frac{z-\propto}{z\bar{\propto}-\rho^2}\right]$ such that $|\propto| < \rho$.
10. Discuss the transformation w =z^n.
11. Discuss the transformation w =z^2.
12. Discuss the transformation w =$\frac{1}{z}$.
13. Discuss the transformation w = e^2.
14. Discuss the transformation w = Sin z.

15. Discuss the transformation w = $\sqrt{z}$.

UNIT - v

1. State & Prove Cauchy – Goursat's theorem.
2. State & Prove Cauchy integral formula.
3. State & Prove Cauchy integral formula for higher order derivatives.
4. State & Prove Gauss mean value theorem.

STATICS – BMA 54

SECTION-A

UNIT-I

1. Define linear momentum.
2. If the resultant forces 3P, 5P is equal to 7P, find the angle between the forces.
3. Define Resultant force.
4. State the types of forces.
5. Define Force.
6. State Hooke's law.
7. Derive an expression for angle between resultant of two forces and one of the force , when they are perpendicular to each other.
8. State the types of forces.
9. Find the resultant that when the two forces acting on a same line.
10. State the parallelogram law of forces.
11. If the resultant of two forces acting at a point with magnitudes 7 and 8 is a force with magnitude 13, find the angle between the two given forces.
12. Define tension.
13. When the two forces are equal in magnitudes, what is the resultant of these forces?
14. Define Weight.
15. Define resultant force.
16. State the converse of the triangle of forces.
17. Define the earth's gravitation.

UNIT-II

18. State the triangle law of forces.
19. Define like parallel forces.
20. Define moment of a vector about a point.
21. Define like unparallel forces.
22. State Lami 's theorem.
23. Define Parallel forces.
24. If three forces keep a particle in equilibrium where these forces lie?
25. Define moment of a vector about a line.
26. What are the conditions of equilibrium of three coplanar parallel forces.
27. State the principle of transmissibility of forces on a rigid body.
28. Define moment of a force about a line.
29. Define Applied force and Effective force.
30. Define Kinetic energy.

UNIT-III

31. Define limiting friction.
32. State any two laws of friction.
33. Define the relation between limiting frictionand normal reaction.
34. Define Couple.
35. Define moment of a couple.
36. What is cone of Friction.
37. Define Friction.
38. Define angle of Friction.
39. Define coefficient of Friction.
40. What is meant by Cone of Friction.

UNIT-IV

41. Define centre of mass.
42. Write down the components of velocity in Cartesian co-ordinates.
43. What is the position vector of mass centre of 3 particles of same mass m?
44. What is the position vector of mass centre for continuous mass distribution of total mass M?
45. What is the centre of mass of triangular lamina?
46. Find the centre of mass of three particles of different masses placed at the vertices of three uniform rods forming a triangle?

UNIT-V

47. Write down the centre of gravity of a right circular cone.
48. Define centre of gravity of two particles of masses$m_1\ and m_2$.
49. Find the centre of gravity of hollow hemisphere.
50. Define centre of gravity of a system of particles.
51. Define centre of gravity of a rigid body.
52. What is the centre of gravity of a circular arc?

53.Find the mass centre of three particles of some mass

SECTION -B

UNIT-I

1. What is equation of motion?
2. Explain the types of forces.
3. State and prove Parallelogram law of forces.
4. State and prove Triangle law of forces.
5. State and prove converse of Triangle law of forces.

6. State and prove Lami's theorem.
7. Show that if three forces keep a particle in equilibrium then the forces are coplanar.
8. Explain Equilibrium of a particle under several forces.
9. Derive the resultant of coplanar forces using their components. Also find the angle between resultant and each force.
10. Find the resultant of two forces on a particle..
11. The resultant that two forces can have is of magnitude R, and the least is of magnitude S. Show that when they act at an angle 2α the magnitude of their resultant is $\sqrt{R^2 cos^2\alpha + S^2 sin^2\alpha}$.
12. Forces of magnitude F_1,F_2,F_3 act on a particle. If their directions are parallel to $\overrightarrow{BC}, \overrightarrow{CA}, \overrightarrow{AB}$where ABC is a triangle , show that the magnitude of their resultant is $\sqrt{F_1^2 + F_2^2 + F_3^2 - 2F_2F_3\cos A - 2F_3F_1\cos B - 2F_1F_2\cos C}$
13. If the resultant of two forces acting at a point with magnitude 7 and 8 is a force with magnitude 13, find the angle between the two given forces.
14. Show that the resultant of forces$\lambda_1\overrightarrow{OA}$, $\lambda_2\overrightarrow{OB}$, is ($\lambda_1$+$\lambda_2$) $\overrightarrow{OP}$, what P divides AB in the ratio λ_1: λ_1.
15. Two forces of magnitude P and Q act at a point. They are such that, if the direction of one is reversed, then the resultant turns through a right angle. Show that P=Q.
16. Three forces of equal magnitude P act on a particle. If their directions are parallel to the sides BC,CA,AB of a triangle ABC, show that the magnitude of their resultant is P$\sqrt{(3 - 2cosA - 2cosB - 2cosC)}$.
17. The greatest resultant that two forces can have is of magnitude R, and the least is of magnitude S. Show that, when they act an angle α, $\sqrt{R^2cos^2\left(\frac{\alpha}{2}\right) + S^2sin^2(\frac{\alpha}{2})}$, is the magnitude of their resultant.
18. Show that the resultant of two forces secB, secC, acting along the sides AB, AC of a triangle ABC is a force (tanB+tanC) acting along AD, where D is the foot of the perpendicular from A on BC.
19. Three forces act along the bisectors of angles A, B, C of a triangle ABC. Show that if the forces are in equilibrium then their magnitudes are in the ratio Cos$\frac{A}{2}$: $Cos\frac{B}{2}$: $Cos\frac{C}{2}$.
20. Find the magnitude and direction of the resultant of three coplanar forces P,2P,3P acting at a point and inclined mutually at an equal angle of $120°$.

UNIT-II

21. What are the conditions following equilibrium of a rigid body.
22. Find the resultant of two like parallel forces acting on a rigid body.
23. Find the resultant of two unlike parallel forces acting on a rigid body.
24. State and prove Varignon's theorem.
25. Show that the moment of a couple is independent of a point about which the moment is obtained.

26. Explain moment of a couple,arm and axis of a couple.
27. Show that a system of coplanar force reduce either to a single force or to a couple.
28. Show that the three coplanar forces represented by and acting along the sides of a triangle taken in order, reduces to a couple.
29. Prove that two couples of equal moments are equivalent.
30. Show that two coplanar couples whose moments are equal in magnitude but opposite in the direction keeps a rigid body in equilibrium.
31. Show that a couple can be transferred to a plane parallel to its own plane without altering its effect on a rigid body on which it is acting.
32. Find an equivalent couple having its constituent forces along two given parallel line in the plane of the couple.
33. State the cotangent formulae.
34. Forces of magnitude 3P,4P,5P act along the sides BC,CA,AB of an equilateral triangle of side A. Find the moment of the resultant about A.
35. One end of the rope of 20 m is to be fixed to a telegraph post and the other end is to be pulled by a man to the ground with the constant force F, to cause the maximum effect to overturn the post, At what height the rope is to be fixed?
36. If two like parallel forces of magnitude P,Q where P>Q acting on a rigid body at A, B are interchanged in position. Show that the line of action of resultant is displaced through a distance $\frac{(P-Q)}{P+Q}AB$.
37. A uniform plank AB of length 2a and weight W is supported horizontally on two horizontal jpegs c and d at a distance 'd' apart. The greatest weight that can be placed at the two ends without upsetting the plank are W_1 and W_2 respectively. Show that $\frac{W_1}{W+W_1}+\frac{W_2}{W+W_2}=\frac{d}{a}$.
38. Three like parallel forces act at the vertices of a triangle ABC if the resultant passes through Ortho centre O. Show that $\frac{P}{tanA}=\frac{Q}{tanB}=\frac{R}{tanc}$.
39. Three like parallel forcesof magnitude P,Q,Ract at the vertices of a triangle ABC.Show that the resultant passes through the circumcentre S of a tiangle then Show that $\frac{P}{sin2A}=\frac{Q}{sin2B}=\frac{R}{sin2C}$.
40. If I is the incentre of triangle ABC. If forces of magnitude P,Q,R acting along the bisectors IA,IB,IC are in equilibrium, show that $\frac{P}{sin2A}=\frac{Q}{sin2B}=\frac{R}{sin2C}$.
41. ABCDEF is a regular hexagon if the forces 2P, P,3P,5P,6P,2P act along AB,BC,DC,ED,EF,AF. Show that six forces are equivalent to the couple and find the moment of the couple.
42. Forces 3, 2, 4, 5 kg. wt. act along the sides AB,BC,CD,DA of a square. Find their resultant and its line of action.

UNIT-III

43. Show that a system of coplanar forces acting on a rigid body can be reduced to a force at an arbitrarily chosen point and a couple in the plane.
44. Discuss the necessary and sufficient conditions for the system of coplanar forces to keep a rigid body in equilibrium.
45. State the laws of Friction.
46. Define Normal reaction and friction with an example.
47. Discuss about limiting friction.
48. Define angle of friction and cone of friction.
49. The uniform ladder AB rest in limiting equilibrium with end A on a rough floor. The coefficient of friction being μ with the other end B against a smooth vertical wall. Show that if θ is a inclination of the ladder to the vertical then prove that tanθ =2 μ.
50. A rod AB whose centre of gravity divides it into two parts of length a and b has its end A and B tied to a string which passes over a smooth fixed peg θ and the rest inclined to a vertical angle θ. Show that cotθ =$\frac{a-b}{a+b}$ $cot\alpha$.
51. A uniform ladder rests with its lower end on a rough ground and its upper end against a smooth vertical wall when the equilibrium is limiting show that the angle which the ladder makes with the wall is 30°.
52. The uniform ladder AB rest in limiting equilibrium with its lower end on a rough ground and its upper end against a rough vertical wall. The coefficient of friction being $\mu\ and\ \mu'$ respectively and if the ladder being on the point slipping at both ends and if θ is the inclination of a ladder to the horizontal then Show that tan θ=$\frac{1-\mu'\mu}{2\mu}$.
53. A uniform rod AB rest with the fixed hemispherical bowl whose radius is equalo to the length of the rod and if the coefficient of friction being μ Show that in limiting equilibrium the inclinationθ of the rod toi the horizontal is given by tanθ=$\frac{4\mu}{3-\mu^2}$.
54. A uniform rod rest in limiting equilibrium with a rough hollow sphere if the rod substend an angle 2α at the centre of the sphere and λbe the angle of friction. Show that the inclination θ of the rod to the horizontal is $\tan^{-1}\frac{sin2\lambda}{cos2\alpha+cos2\lambda}$.
55. A solid hemisphere rests on a rough horizontal plane and against a smooth vertical wall. Show that if the coefficient of friction μ is greater than 3/8, then the hemispher can rest in any position and if it is less, the least angle that base of hemisphere makes with the vertical is $\cos^{-1}(\frac{8\mu}{3})$.
56. A ladder which stands on a horizontal ground leaning against a vertical wall, has its centre of gravity at a distances a and b from its lower and upper ends resp. Show that if the ladder is in limiting equilibrium and its $\mu$and μ' are the coefficients of friction at the lower and upper contacts, its inclination θ to the vertical is given by tan θ=$\frac{(a+b)\mu}{(a-b)\mu\mu'}$\

57. Show that the greatest inclination of a rough inclined plane to the horizon so that a particle will remain on it at rest, is equal to the angle of friction.
58. The uniform ladder AB rest in limiting equilibrium with its lower end on a rough ground and its upper end against a rough vertical wall. The coefficient of friction being μ, Show that if θ is the inclination of the ladder to the vertical is tan θ=2μ.

UNIT-IV

59. Find the mass centre of triangular lamina.
60. Find the mass centre of three particles of same mass placed at the vertices of a triangle.
61. Find the mass centre of three particles of same mass placed at the midpoint of the sides of a triangle.
62. Find the mass centre of three particles of certain mass placed at the vertices of a triangle.
63. Find the mass centre of triangle formed by three uniform rods.
64. Find the mass centre of the lamina in the form of a trapezium.
65. Show that the mass centre of mass P,Q,R placed at the vertices A, B, C, of a triangle is at a distance from the sides BC,CA,AB proportional to $\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}$.
66. Let ABCD be a trapezium in which AB,CD are parallel and of length a and b. Show that its distance of its mass centre from AB is $\frac{h(a+2b)}{3(a+b)}$ where h is the distance between AB and CD.
67. A rod of length 5a is bent so as to form five sides of a regular hexagon. Show that its centre of mass is at a distance a $\sqrt{1.33}$ from either end of the rod.

UNIT-V

68. Find the mass of a thin wire in the form of a circular arc.
69. Find the centre of mass of the lamina in the form of a sector of a circle.
70. Find the mass centre of the cardiodal lamina.
71. Find the centre of gravity of a quadrant of an elliptic lamina.
72. Find the centre of gravity of a sphere of radius a cut off by a plane at a distance c from thecentre.
73. Find the centre of mass for a hollow right circular cone of height h.
74. Find the mass centre of a non-homogenous solid.
75. If OA and OB are two uniform rods of lengths 2a, 2b. If <AOB=α, Show that the distances of the mass centre of the rods from O, is $\frac{(a^2+2a^2b^2cos\alpha+b^2)}{a+b}$.

10-MARK QUESTIONS

UNIT-I

1. Two forces of magnitudes F_1 and F_2 act at a point. They are inclined at an angle α. If the forces are interchanged, show that their resultant is turned through angle $2(tan^{-1}(\frac{F_1-F_2}{F_1+F_2}tan\frac{\alpha}{2})$.
2. The magnitude of the resultant of two given forces of magnitude P and Q is R. If Q is doubled, then R is doubled. If Q is reversed, then also R is doubled. Show that P:Q:R=$\sqrt{2}$:$\sqrt{3}$:$\sqrt{2}$.
3. Three forces proportional to the sides of a triangle act at the vertices towards and perpendicular to the corresponding sides. Prove that the forces are in equilibrium.
4. The resultant of two forces P,Q is of magnitude P. Show that if P is doubled , the new resultant is perpendicular to the force Q and its magnitude is $\sqrt{4P^2-Q^2}$.
5. Forces of magnitude 2, $\sqrt{3}$, 5,$\sqrt{3}$,2 respectively act at one of the angular points of a regular hexagon towards the other five points in order. Show that their resultant is of magnitude 10 and makes an angle of 60° with the first force.
6. The resultant of two forces of magnitudes P and Q acting at a point, has magnitudes (2n+1)$\sqrt{P^2+Q^2}$ and (2n-1)$\sqrt{P^2+Q^2}$ when the forces are inclined at $\propto$ and $90°-\propto$ respectively. Show that tan$\propto$= $\frac{n-1}{n+1}$.
7. Three forces of equal magnitude P act on a particle. If their direction are parallel to the sides BC,CA, AB of a triangle ABC. Show that the magnitude of their resultant is P$\sqrt{3-cosA-2cosB-2cosC}$
8. A heavy carriage wheel of weight ω and radius r is to the dragged over on obstacle of height h by a horizontal force of magnitude F applied to the centre of the wheel. Show that F must be slightly greater then $\omega\frac{\sqrt{2rh-h^2}}{(r-h)}$.
9. Find the magnitude and direction of the resultant of F_1,F_2 acting at a particle.

UNIT-II

10. Find the resultant of the two like parallel forces acting on a rigid body.
11. State and prove Lami's theorem.
12. If the coplanar forces keep a rigid body in equilibrium then either they all are parallel to one another or they are concurrent.
13. State and prove Varignon's theorem.
14. A weight is supported on a smooth plane of inclination α by a string inclined to the horizontal at angle γ. If the slope of the plane be increased to β,and the slope of the string be unaltered, the tension of the string is doubled. Prove that cot$\alpha-2cot\beta=tan\gamma$.

15. Find the resultant of two –parallel forces acting on a rigid body.

UNIT-III

16. ABCD is a uniform square plate of side 2a resting with its plane vertical with the side AB on rough inclined plane of angle α. C is the highest point of the plate. A gradually increasing force is applied at C horizontally. Prove that if λ is the angle of friction, then the plate tilts if 1+tanα<2tan(α+λ).
17. Find the least force required to drag a particle on a rough horizontal plane and show that the least force acts in a direction making with the horizontal, an angle equal to the angle of friction.
18. A uniform rod AB rests within affixed hemispherical bowl whose radius is equal to the length of the rod. If μ is the coefficient of friction between the rod and the bowl, show that, in limiting equilibrium, the inclination θ of the rod to the horizontal is given by tan$\theta = \frac{4\mu}{3-\mu^2}$.
19. A square lamina rests with the ends of a side against a rough vertical wall and a rough horizontal ground. If the coefficient of friction for the ground and the wall are μ and μ' show that when the lamina is one the point of slipping the inclination of the side in question to the horizontal is$tan^{-1}(\frac{1-\mu\mu'}{1+2\mu+\mu\mu'})$.
20. A uniform rod rests in equilibrium with in a rough hollow sphere. If the rod subtends an angle (2α) at the centre of the sphere and if λ is the angle of friction, show that the inclination θ of the rod to the horizontal is $tan^{-1}(\frac{sin2\lambda}{cos2\alpha+cos2\lambda})$.

UNIT-IV

21. A particle is placed on a rough plane inclined at an angle α to the horizontal.If the force, which acting parallel to the plane in the upward sense, is just sufficient to keep the particle at the point of moving up the plane, is n times the force, which acting in the same manner, is just sufficient to keep the particle at the point of moving down the plane. Show that$tan\alpha = \frac{\mu(n+1)}{(n-1)}$.

22. A ladder length l rests on a rough horizontal grand with its upper and projecting slightly over a smooth horizontal rod at a height 'h' above the ground. If the ladder is about to slip, show that the coefficient of friction is equal to $\frac{h\sqrt{l^2-h^2}}{l^2+h^2}$.

23 .State the five laws of frictions.

24. A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall. The ground and the wall are both rough, the coefficient of friction being μ and μ'respectively. If the ladder is on the point of slipping at both ends prove that the inclination

of the rod to the horizontal is $tan^{-1}(\frac{1-\mu}{2\mu})$

25. A solid hemisphere rests on a rough horizontal plane and against a smooth vertical wall. Show that, if the coefficient of friction, μ is greater then 3/8 then the hemisphere can rest in any position and if it is less thenleast angle that the base of the hemisphere can make with the vertical is $cos^{-1}(\frac{3\mu}{8})$.

26 .Find the distance of the mass centre of a hemispherical shell from the centre.

27. Find the mass centre of

(a) Hollow right circular cone of height h.
(b) Hollow hemisphererical

UNIT-V

1. Find the centre of mass of three uniform rods forming a triangle.
2. Find the centre of gravity of a hollow hemisphere.
3. Find the centre of gravity of a trapezium.
4. Find the centre of gravity of a solid hemisphere.
5. Find the centre of gravity of a solid hemisphere of radius a.
6. Find the mass of a solid right circular cone of height h.
7. Find the centre of gravity of a hemispherical shell.
8. Find the centroid of a cardioidal lamina.

DYNAMICS BMA 55

UNIT – I

Section -A

1. Define relative velocity.
2. Write the relation between work and power.
3. When will the force field is said to be conservative?
4. Define angular velocity
5. Write any two equations of motion of a particle with constant acceleration.
6. Define acceleration.
7. Define work and power.
8. Write the principles of conservation of mechanical energy.
9. Find the magnitude and direction of the resultant of the velocities $\overline{v_1}, \overline{v_2}$.
10. If $\overline{v_1}$ & $\overline{v_2}$ are of equal magnitude, say v then prove that $|\overline{v_1} + \overline{v_2}| = 2v\, cos\frac{\alpha}{2}$.
11. If $\overline{v_1}$ & $\overline{v_2}$ are perpendicular to each other then choosing $\bar{\imath}$ & $\bar{\jmath}$ in their directions.
12. Express the velocity $\bar{v}$ in terms of its components in two perpendicular directions.
13. A particle has two velocities $\overline{v_1}$ & $\overline{v_2}$. Its resultant velocity is equal to $\overline{v_1}$ in magnitude. Show that when the velocity $\overline{v_1}$ is doubled, the new resultant is perpendicular to $\overline{v_2}$.
14. A point possesses velocities represented by AB & AC two sides of a triangle. Show that its resultant velocity is represented by 2AM, where M is the midpoint of bc.
15. A boat which steam in still water with a velocity of 48 km/h is steaming with its bow pointed due east when it is carried by a current which flows northward with a speed of 14 km/h. Find the actual distance it would travel in 12 minutes.define rectilinear motion.
16. Define relative angular velocity.
17. Find the resultant of velocities of 400 cm. per sec. and 300 cm.per sec. iinclined at an angle of 60°.
18. Find the angle between two velocities of 3 cm/sec. and 5 cm/sec. if their resultant is 7cm/sec.
19. Find in magnitude and direction, the resultant of two equal velocities of 15 km/h.
    i) at 120° to each other ii) at 60° to each other.
20. Resolve a velocity of 100 cm/sec. into two equal velocities at an
    angle of 60° to each other.

Section - B

21. A point possesses four simultaneous velocities whose measure are 1,2,3√3 and 4 respectively. The angle between the first and second is 60°, between the second and third 90° and between the third and the fourth 150°. Find the magnitude and direction of the resultant velocity.

22. A man can swim directly across a stream of breadth 100metres in 4 minutes when there is no current and in 5 minutes when there is a current. Find the velocity of the current.
23. To a man walking at the rate of 4km/h. rain appears to fall vertically. If its real velocity is 8 km/h, find its real direction.
24. To a man walking at 4 km/h along a road running due west, wind appears to blow from the south, while to a cyclist travelling in the same direction at 8 km/h. it appears to come from southwest. What is the true direction and velocity of the wind?
25. Two cars A and B are moving due north and due east at 40 and 30 km/h respectively. At noon B is west of A, at a distance of 20km. when are the cars closest to each other and what is the distance between them at that time?
26. To a cyclist riding due west at 10km/h, the wind appears to him to blow from south. When he doubles his speed, it appears to him to blow from southwest. Show that the speed of the wind is $10\sqrt{2}$km/h and it is from southeast.
27. The total mass of a train is 500 tones. When it moves with a uniform speed of 72 km/h on the level, the resistance due to friction is 16kg.weight per tone. Find the power developed then.
28. Show that the angular velocity about a fixed point A of a particle P moving uniformly in a straight line varies as the square of the distance of the line from the fixed point.

SECTION - C

29. The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. Show that the velocity with which the middle point of the train passes the same point is $\sqrt{\frac{u^2+v^2}{2}}$.
30. A train goes from rest at one station to rest at another station 3 km. off, being uniformly accelerated for the first two-thirds of the journey and uniformly retarded for the remainder and takes 3 minutes to describe the whole distance. Find the acceleration, the retardation and the maximum velocity.
31. An express train moving at 60 m/sec. reduces its speed to 20 m/sec in a distance of 480 metres. At what distance will the train come to a stop? If the brake power be increased by 12½ %, Show that the train will stop in a distance of 480 metres.
32. A point moves with uniform acceleration and v_1, v_2, v_3 denote the average velocities in three successive intervals of time t_1, t_2, t_3. Prove that $v_1-v_2:v_2-v_3 = t_1+t_2 : t_2+t_3$.
33. Verify the principle of conservation of energy in the case of a particle sliding down a smooth inclined plane.
34. Find the power required to pump $6m^3$ of water per minute from a depth of 20m and deliver it through a pipe of cross sectional area $0.004m^2$
[The mass of $1m^3$ of water is 10^3kg.]

35. A boat is rowed with a velocity of 4 m/sec. and directed straight across a river flowing at 3 m/sec. if the breadth of the river be 500 m, find how far down the river the boat will reach the opposite point.

UNIT-II - PROJECTILES

Section - A

1. Define horizontal range of a projectile.
2. Give the formula for finding the time taken to reach the topmost point of a projectile.
3. Define the velocity of projection.
4. Define time of flight.
5. Define the angle of projection.
6. Define trajectory of a projectile
7. Write the maximum horizontal range of a projectile.
8. State the equation of the trajectory
9. A projectile is thrown with a velocity of 20m/sec. at an elevation 30°. Find the greatest height attained and the horizontal range.
10. A particle is projected with a velocity of 9.6 metres at an angle of 30°. find i) the time of flight ii) the greatest height of the particle.
11. Find the velocity and direction of a shot which passes in a horizontal direction over the top of wall 100m way and 50m high.
12. A projectile reaches a maximum height of 75m and is then travelling with a velocity of $10\sqrt{\frac{3g}{2}}$ m/sec. find the angle of projection and the horizontal range.
13. If the range on the horizontal plane is equal to the height to which the velocity of projection is due, show that the angle of projection is either 15° or 75°.
14. A body projected with the same velocity at two different angles covers the same horizontal range R. It T_1,T_2 be the two times of flight, prove that $R = \frac{1}{2}gT_1T_2$.
15. Prove that $gT^2 = 2Rtan\alpha$, where T is the time of flight R, the horizontal range and α, the angle of projection of a particle projected from the ground.
16. If the range on the horizontal plane through the point of projection and the greatest height above the point of projection are R and H, show that the velocity of projection is $\sqrt{2gH + \frac{gR^2}{rH}}$.
17. The greatest range with a given velocity of projection on a horizontal plane is 3000metres. Find the greatest ranges up and down the plane inclined at 30° to the horizon.
18. If the greatest range down the inclined plane is thrice the greatest range up the plane, show that the angle of inclination of the plane is 30°.
19. A stone is projected horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. Find twhere it will strike the elvel ground through the foot of the tower.

20. A is a point on the ground, O is a point above A such that AO=h, A particle projected horizontally from O hits the ground at B, to find the time of flight T and the range AB.

Section - B

21. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If A, B are the base angles, and α the angle of projection. Show that tan α = tan A + tan B.
22. Show that the path described by a particle is projected horizontally from a point at a certain height above the ground is a parabola.
23. Derive for a projectile $R = \frac{u^2 - sin2\alpha}{g}$.
24. Show that the greatest height reached by a projectile whose initial velocity is V and angle of projection is α is unaltered if V is increased to kV and α is decreased by λ where cosec λk (cot λ – cot α).
25. If t is the time of flight, R the horizontal range and α the angle of projection, Show that $gT^2 = 2Rtan\alpha$. If α = 60° find interms of R, the heiht of the projectile when it has moved through a horizontal distance equal to $\frac{3R}{4}$.
26. A particle is projected so as to graze the tops of two walls each of height 7 metres at distance of 10 metres and 50 metres respectively from the point of projection. Find the angle of projection.
27. If h and h' be the greatest heights in the two paths of a projectile with a given velocity for a given range R, Prove that $R = 4\sqrt{hh'}$.
28. A particle is projected with a velocity of 49 m/sec. at an angle of 60° to the horizontal. Fond after what time it moves at an angle of 45° to the horizontal.
29. In the case of a projectile, verify that kinetic energy + potential energy = a constant.
30. Discuss the maximum range of a projectile on an inclined plane.
31. A particle is projected from a given point on the ground, just clears a wall of height h at a distance a from the point of projection. If the particle moves in a vertical plane perpendicular to the wall and if the horizontal range I R, Show that the elevation of projection is given by $\tan\alpha = \frac{Rh}{a(R-a)}$.
32. A particle is projected from a point P at an angle of 45° to the horizontal. If PQ is the horizontal range and if angles of elevation of the particle at P and Q at any instant of flight are α & β respectively. Show that tan α + tan β = 1.
33. Show that the greatest range of a projectile on an inclined plane through the point of projection is equal to the distance through which the particle could fall freely during the time of flight.
34. A particle is to the projected from a point P so as to pass through another point Q. Show that the product of the two times of flight from P to Q with a given velocity of

projection is $2\frac{PQ}{g}$.

35. Show that the velocity at any point P of a projectile is equal I magnitude to the velocity acquired in falling freely from the directrix to the point.

Section – C

36. If v_1, v_2 be the velocities of a projectile at the end of focal chord of its path and v is the velocity at the vertex, Prove that $v_1^{-2} + v_2^{-2} = v^{-2}$.
37. In projectiles a) Find the greatest height attained b) Find the time of flight.
38. Show that the path of a projectile is parabola.
39. A particle is projected from a point O on a plane of inclination β with velocity u making an angle α with the horizontal. Find the range on the plane.
40. A ball is projected so as just to clear two walls, the first of height 'a' at a distance 'b' from the point of projection and the second of height 'b' at a distance 'a' from the point of projection. Show that the range on the horizontal plane is $\frac{a^2+ab+b^2}{a+b}$ and the angle of projection exceeds $tan^{-1}3$.
41. Show that the greatest height which a particle with initial velocity v can reach on a vertical wall at a distance 'a' from the point of projection is $\frac{v^2}{2g} - \frac{ga^2}{2v^2}$. Prove also that the greatest height above the point of projection attained by the particle in its flight is $\frac{v^6}{2g(v^4+g^2a^2)}$.
42. Show that for a given velocity of projection the maximum range down an inclined plane of inclination α bears to the maximum range up the inclined plane the ratio $\frac{1+sin\alpha}{1-sin\alpha}$.
43. A particle is projected at an angle α with a velocity u and it strikes up an inclined plane of inclination β at right angles to the plane. Prove that
$i)\cot\beta = 2\tan(\alpha-\beta)$ $\quad$ $ii)\cot\beta = \tan\alpha - 2\,tan\beta$
If the plane is struck horizontally, Show that $\tan\alpha = 2\,tan\beta$.

44. A particle is projected with velocity $\sqrt{2ga}$ from a point at a height h above a level plain. Show that the tangent of the angle of elevation for maximum range on the plain is $\sqrt{\frac{a}{a+h}}$ and the maximum range is $2\sqrt{a(a+h)}$.
45. Show that for a given initial velocity of projection there are, in general two possible directions of projections so as to obtain a given horizontal range.
46. Two particles are projected at an interval of time t from the same point with velocities u & u' at elevation α & α'. If the particles collide, Show that

$t = \frac{2u\,u'\sin(\alpha-\alpha')}{g(u\cos\alpha + u'\cos\alpha')}$.

47. Show that for a given range on a given inclined plane there are two directions of projection which are equally inclined to the direction of projection to get the maximum range.

48. A particle projected with a speed u strikes at right angles a plane, through the point of projection, inclined at an angle β to the horizon, If α,T,R are the angle of projection, the time of fight and the range on the inclined plane. Show that $T = \frac{2u}{g\sqrt{1+3sin^2\beta}}$ and $R = \frac{2u^2 sin\beta}{g(1+3sin^2\beta)}$.

49. A particle projected from the top O of a wall AO, 50 m height, at an angle of 30° above the horizon, strikes the level ground through A at B at an angle of 45°. Show that the angle of depression of b from O is $tan^{-1}\frac{\sqrt{3}-1}{2\sqrt{3}}$.

50. P is a point at a horizontal distance a and a vertical distance b from the point of projection. It is required to projected a particle to pass through P, with an initial velocity V. Show that this is impossible if $V^2 < g(b + \sqrt{a^2+b^2})$ and that, if $V^2 > g(b + \sqrt{a^2+b^2})$ there are two possible directions of projection.

Unit – III IMPACT

Section – A

1. Define the impulsive force
2. State the principle of conservation of linear momentum.
3. State Newton's experimental law.
4. Define the direct impact of two spheres.
5. If a body is perfectly elastic, what is the coefficient of elasticity *e* of the body?
6. A particle ball drops from the ceiling of a room and after rebounding twice from the floor reaches a height equal to one half that of the ceiling. Show that the coefficient of restitution is $(1/2)^{1/4}$
7. A particle falls from a height *h* in time *t* upon a fixed horizontal plane. Prove that it rebounds and reaches a maximum height e^2h in time *et.*
8. A ball is projected with a velocity of 24√3 ft/sec. at an elevation of 45˚. It strikes a wall at a distance of 18 ft. and returns to the point of projection. Show that *e = 1/2.*
9. A ball fall from a height *h* in time *t* upon a fixed horizontal plane. Prove that it rebounds and reaches the maximum height e^2h in time et loss in K.E in the hit is $mgh(1-e^2)$.
10. A ball falls from a height of 64cm on a smooth horizontal plane. Of the coefficient of restitution is ½, find the height to which it rises after rebounding four times.

11. An ivory ball falling from a height of 100 cm. rises to a height of 46 cm. after rebounding twice. Show that $e = \left(\frac{23}{50}\right)^{\frac{1}{4}}$.
12. A & B are two perfectly elastic equal balls. B is at rest and is struck obliquely by A. Show that after impact their directions are at right angles.
13. A ball dropped from a height h on a horizontal plane bounces up and down. If the coefficient of restitution is e, Prove that the whole distance H covered before it comes to rest is $h\frac{1+e^2}{1-e^2}$.
14. Two equal balls of mass m are in contact on a table. A third equal ball strikes both symmetrically and remains at rest after impact. Show that e = 2/3.
15. Show that when two spheres of equal masses m collide directly the velocities of the spheres are interchanged if e = 1.

Section – B

16. Two equal marble balls A, B lie in a horizontal circular groove at the opposite ends of a diameter; A is projected along the groove and after time t, impinges on B; Show that a second impact takes place after a further interval $2t/e$.
17. A smooth circular table is surrounded by a smooth rim whose interior surface is vertical. Show that a ball projected along the table from a point A on the rim in a direction making an angle α with the radius through A will return to the point of projection after two impacts if $\tan\alpha = \frac{e^{3/2}}{\sqrt{1+e+e^2}}$.
18. Find the loss of Kinetic energy due to direct impact of two smooth spheres.
19. An elastic sphere is projected from a given point with velocity v at an angle inclination α to the horizontal and after hitting a smooth vertical wall at a distance d from the point of projection returns the point . Prove that $d = \frac{v^2}{g} sin2\alpha \frac{e}{1+e}$.
20. Find the velocities of two smooth spheres after a direct impact between them.
21. A particle falls from a height h upon a fixed horizontal plane; if e be the coefficient of restitution, show that the whole distance described before the particle has finished rebounding is $h\frac{1+e^2}{1-e^2}$. Also show that the whole time taken is $\frac{1+e}{1-e}\sqrt{\frac{2h}{g}}$.
22. If two equal and perfectly elastic spheres impinge directly, show that they interchange their velocities.
23. A smooth sphere impinges on another at rest. After the impact their directions are at right angles. Show that if they are assumed to be perfectly elastic, their masses are equal.

24. Two spheres of equal mass moving in the same straight line with velocities u, u' collide and rebound, the coefficient of restitution being ½. Prove that exactly half the energy is lost in collision if (1-√2)u = (1+√2) u'.

25. A shell of mass M is moving with velocity v. An interval explosion generates an energy E and breaks the shell into two portions whose masses are in the ratio a:b. The fragments continue to move in the orginal line of motion of the shell. Show that their velocities are $v+\sqrt{\frac{2be}{aM}}$ and $v-\sqrt{\frac{2ae}{bM}}$.
26. A shot of mass m is discharged from a gun of mass nm with a relative velocity V. Find the velocities of the shot and the gun and show that the total K.E generated is $\frac{1}{2}\frac{nm}{n+1}V^2$.
27. A smooth sphere of mass m impinges obliquely on a smooth sphere of mass M which is at rest. Show that if m = eM, the directions of motion after impact are at right angles.
28. There are two equal perfectly elastic balls. One is at rest and is struck obliquely by the other. Show that after impact their directions of motion are at right angles.
29. A sphere impinges on a sphere of equal mass which is at rest; if the directions of motion after impact be inclined at angles of 30˚ to the original direction of motion of the impinging sphere, Show that the coefficient of restitution is 1/3.

Section – C

30. A gun of mass M fires a shell of mass m, the elevation of the gun being α. If the gun can recoil freely in the horizontal direction, Show that the angle θ which the path of the shell initially makes with the horizontal is given by the equation $tan\theta = \left(1+\frac{m}{M}\right) tan\alpha$.
31. Find the loss of K.E due to oblique impact of two smooth spheres.
32. A particle is projected from a point on an inclined plane and at the r^{th} impact it strikes the plane perpendicularly and at the n^{th} impact is at the point of projection. Show that $e^n - 2e^r + 1 = 0$.
33. Find the velocities of two smooth sphere after oblique impact between them.
34. When two smooth spheres collide directly, find the impulse imported to each sphere and the change in the total K.E of the sphere.
35. The masse of three spheres A, B, C are 7m, 7m, m their coefficient of restitution is unity. Their centres are in a straight line and C lies between A and B. Initially A & B are at rest and C is given velocity in the line of centres in the direction of A. Show that it strikes A twice and B once, and that the final velocities of A, B, C are

21 : 12 : 1.

36. Two spheres A & B of same size and of masses 2 kg and 30 kg. respectively lie on a smooth floor so that their line of centres is perpendicular to a fixed vertical wall, A being nearer to the wall. A is projected towards B. Show that , if the coefficient of restitution between the two spheres and that between the first sphere and the wall is 3/5, then A will be reduced to rest after its second impact with B.
37. A gun of mass M fires a shell of mass m horizontally and the energy of the explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of the horizontal recoil V of the gun is given by $V^2 = \frac{2m^2gh}{M(m+M)}$.
38. a) An elastic ball falls through a height h and impinges at A on a smooth plane inclined at an angle β to the horizontal. Show that if the ball impinges on the inclined plane again at B and if e is the coefficient of restitution , then AB = 4he(e+1)sinβ

    b) if the ball falls through a height 20m. and if the plane is inclined at 30° to the horizontal and if the ball descends to the bottom B in 3 jumps which is 12m. below the point of hitting, along the plane, show that $e (1 + e) (1 + e^2) (1 + e + e^2) = 0.3$.

39. A body of mass m rests on a smooth table. Another mass M moving with velocity U collides with it. Both are perfectly elastic and smooth. The body m is direction in a direction at an angle α to the previous line of motion of the body M. Show that the velocity of m is $\frac{2M}{M+m} u\cos\alpha$.

UNIT-IV CENTRAL ORBITS

Section – A

1. Define central force.
2. Define centre of force.
3. Define central orbit.
4. Define Apse.
5. Write the formula for the central orbit in polar coordinates.
6. Define maximum and minimum angular velocity.
7. Define the areal velocity of a particle.
8. Define law of a central force.
9. Define Equiangular spiral.
10. If a point moves so that its radial velocity is k times its transverse velocity, Show that its path is an equiangular spiral.
11. Show that a central orbit is a plane curve.
12. Show that the force towards the pole under which a particle describes the curve $r^n = a^n \cos n\theta$, varies inversely as the $(2n+3)^{th}$ power of the distance from the pole .
13. Write down the kepler's first law.
14. Write the equation for an attractive central force.
15. Write the differential equation of a central orbit in p-r coordinates.

16. If the radial & transverse velocities of a particle are always proportional to each other, find the equation of its path.

Section-B

17. The velocities of a particle along an perpendicular to the radius vector are λr and $\mu\theta$. Find the path and show that the acceleration components along perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}, \mu\theta\left(\lambda + \frac{\mu}{r}\right)$.
18. The velocities of a particle along and perpendicular to the radius from a fixed origin are a & b. Find the path and the accelerations along and perpendicular to the radius vector.
19. Show that the path of a point P whose velocity is such that is components in a fixed direction and in the direction perpendicular to the line joining P to a fixed point O are respectively the constants u & v is a conic with a focus at O and eccentricity u/v.
20. The velocities of a particle along and perpendicular to the radius vector are λr^2 & $\mu\theta^2$ where μ & λ are constants. Show that the equation to the path of the particle is $\frac{\lambda}{\theta} + c = \frac{\mu}{2r^2}$, where C is a constant.
21. A particle describes the equiangular spiral $r = ae^{\theta cot\alpha}$ whose pole is O. If its radial velocity at distance r from O is k/r(k is constant), prove that the acceleration of the particle towards O is $\frac{k^2 sec^2\alpha}{r^3}$.
22. Obtain the differential equation of central orbit in polar coordinates.
23. Obtain the differential equation of a central orbit in p –r coordinates.
24. Find the orbit of a particle moving under an attractive force varying as the distance.
25. A particle moves along the path $r = e^{\theta}$ under a central force. Show that the force is $\frac{2mh^2}{r^3}$ and speed of the particle is $\frac{h}{r}\sqrt{2}$.
26. A particle describes a circular orbit under an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of the distance.
27. A particle moves with a central acceleration $\mu[3au^4 - 2(a^2 - b^2)u^5]$ being projected from an apse at a distance a + b with a velocity $\frac{\sqrt{\mu}}{a+b}$. Show that the equation of its orbit is $r = a + b\cos\theta$.
28. A particle describes an elliptic orbit under a central force towards one focus s. If v_1 is the speed at the end B of the minor axis and v_2, v_3 speed at the ends A, A' of the major axis, show that $v_1^2 = v_2 v_3$.
29. A point P describes an equiangular spiral with a constant angular velocity about the pole O. Show that its acceleration varies as OP and is in a direction making with the tangent at P the same constant angle the OP makes.
30. State the kepler's laws of planetary motion.
31. Show that the motion of particle under a central force is co-planar.
32. Find the law of force to an internal point under which a body will describe a circle.
33. If the radial & transverse velocities of a particle are always proportional to each other, show that the equation of the path is of the form $r = A e^{k\theta}$ where a & k are constants.

34. If the angular velocity of a particle about a point in its plane of motion be constant, prove that the transverse component of its acceleration is proportional to the radial component of its velocity.
35. A particle moves in a curve under a central attraction so that its velocity at any point is equal to that in a circle at the same distance and under the same attraction. Show that the path is an equiangular spiral and that the law of force is that of the inverse cube.

Section – c

36. Show that the velocity of a particle moving in an ellipse about the centre of force at a focus is compounded of two constant velocities, namely, a) μ/h perpendicular to the radius vector
 b) eμ/h perpendicular to the major axis.
37. Find the law of force and the sped of the particle when the central orbit is a conic with the centre of force at one focus.
38. Find the tangential and normal acceleration of particle moving along a curve.
39. A particle moves in an ellipse under a force which is always directed towards its focus. Find the law of force, the velocity at any point of the path and its periodic time.
40. Find the orbit of a particle moving under an attractive central force varying inversely as the square of the distance.
41. A particle moves with a central acceleration μr^{-7} and starts from an apse at a distance a with a velocity equal to the velocity which would be acquired by the particle travelling from rest at infinity to the apse. Show that the equation of its orbit is $r^2 = a^2 \cos 2\theta$.
42. When a central orbit is a conic with the centre of the force at one focus, to find the law of force and the speed of the particle.
43. A particle moves in the path given by the equation $r = a\, e^{\theta}$ with no force in the line joining the particle to the pole. Show that i) The angular velocity of the pole is constant.
 ii) The speed varies as its distance from the pole
 iii) The acceleration is directly proportional to r in magnitude.
44. A particle moves with a uniform speed v along a cardioid $r = a(1+\cos\theta)$. Show that its angular velocity about the pole and the radial acceleration components are $\frac{v}{2a} sec\frac{\theta}{2}$, $\frac{-3v^2}{4a}$.
45. A particle moves with a central acceleration $\mu[3au^4 - 2(a^2 - b^2)u^5]$ being projected from an apse at a distance a + b with a velocity $\frac{\sqrt{\mu}}{a+b}$. Show that the equation of its orbit is $r = a + b\cos\theta$.

UNIT – V MOMENT OF INERIA

Section – A

1. Find the moment of inertia of the parabolic lamina about l
2. Find the moment of inertia of a circular ring of radius a
3. State parallel axis theorem.
4. What is the M.I of circular disc about a diameter.
5. Define the radius of gyration.
6. Define moment of inertia.
7. State perpendicular axis theorem.
8. Define M.I of a particle of mass m about a line.
9. What is the M.I of a solid sphere radius ‘a’ about a diameter.

10. Find the M.I of a parabolic lamina about a line passing through its vertex and perpendicular to its axis.
11. What is the M.I of the parabolic lamina about l, the tangent at the vertex.
12. Find the M.I of a square about its diagonal of length.
13. Find the M.I of a right circular hollow cylinder.
14. Find the M.I of a lamina in the form of a quadrant of an ellipse, about the major axis.
15. Find the M.I of a solid right circular cone.

Section – B

16. Find the M.I of spherical shell about its diameter.
17. State and prove Parallel axis theorem.
18. Find the M.I of a uniform rod about a line through its centre.
19. State & prove Perpendicular axis theorem.
20. Find the M.I of a uniform rectangular parallelepiped.
21. Find the M.I. of a right circular cone.
22. Find the M.I. of a triangular lamina AB with altitude AD = P about its side BC.
23. Find the M.I. of spherical shell about its diameter.
24. Show that the M.I. of a rectangular lamina of mass M and sided 2a & 2b about a diagonal is $M\frac{2a^2b^2}{3(a^2+b^2)}$.
25. Show that the M.I of a triangular lamina of mass M about a side is $Mh^2/6$ where h is the altitude from the opposite vertex.
26. Show that the M.I of an isosceles right angled triangle about its hypotenuse whose length is ‘a’ is$Ma^2/24$.
27. Obtain the M.I of a uniform circular plate about a line perpendicular to the plane of the lamina and at a distance c from the centre.
28. Show that the M.I of hollow sphere whose external and internal radii are a & b about a diameter is $\frac{2M}{5}\left(\frac{a^5-b^5}{a^3-b^3}\right)$. Deduce the M.I of a hollow sphere of radius a.
29. If M is the mass of a right circular cone whose base radius is r and height is h. Show that its M.I about the line through the centre of gravity perpendicular to its axis is $\frac{3\mu}{80}(h^2+4r^2)$.
30. Find the M.I of a circular lamina.

Section – C

31. Find the M.I of a square lamina of side l about one of its diagonals, the density at any point varying as the square of its distance form this diagonal.
32. Find the M.I of a hollow cylinder about its axis.
33. Find the M.I of a solid right circular cone.
34. Find the M.I of a solid right circular cone of base radius ‘a’ about the axis of the cone.
35. Find the M.I for a hollow sphere about its diameter.
36. Show that the M.I about the x –axis of the portion of the parabola $y^2 = 4ax$ bounded by the x – axis and the latus rectum, supposing the surface density at each point to vary as the cube of the abscissa is $12Ma^2/11$, where M is the mass of the lamina.
37. From a circular disc of radius r, a circular portion whose diameter is a radius of the disc is removed. Show that the radius of gyration of the remainder about the common tangent is 5r/4.

38. A uniform lamina of mass M is in the shape of a square ABCD with an external semi-circle drawn on one side AB. Prove that the M.I of the lamina about CD is $\frac{M(64+17\pi)}{128+16\pi}$.
39. Show that the M.I of a paraboloidal solid of revolution of mass M & base radius r about its axis (the solid being generated by revolving a parabolic segment about its axis) is $Mr^2/3$.

GRAPH THEORY - BEMA 56A

2 marks

1. Define a graph and give an examples.

2. Define Multi graph.

3. What is a loop?

4. Define Pseudo graph.

5. Define totally disconnected or null graph.

6. What is labeled graph?

7. Define bi graph or bipartite graph.

8. Define complete bi graph.

9. Define degree of a vertex.

10. What is called isolated point?

11. Prove that the sum of the degree of the products of a graph is twice the number of lines.

12. What is regular graph?

13. When the regular graph becomes complete graph?

14. Define a cubic graph.

15. Define sub graph.

16. which graph is called a spanning subgraph?

17. Define induced subgraph?

18. Define isomorphism of graphs.

19. What is automorphism?

20. Define complementary and self complementary graphs.

21. Define convering and independent set of a graph G.

22. Define minimum covering and maximum independent set.

23.Prove that a set $S \subseteq V$ is an independent set of G iff V-S is a covering of G.

24. Define intersection of graphs.

25. Define line graph.

26. What is adjacency matrix?

27. Define incidence matrix.

28. Define walk, trail & path of a graph.

29. Define a cycle.

30. Define connected graph.

31. Define components of a graph.

32. Define distance between two points

33.Which point is called a cut point?.

34. Which line is called a bridge?.

35. Define a block.

36. Define connectivity of a graph.

37. If G is k-connected graph then $q \geq p .k/2$.

38. Prove that there is no 3-connected graph with 7 edges.

39. Define Eulerian graph.

40. Define Hamiltonian graph.

41. What is theta graph?

42. Define an a cyclic graph.

43. Define a tree.

44. Define a forest.

45. Prove that every non-trivial tree G has atleast two vertices of degree1.

46. What is central of a tree ?

47. Define eccentricity of a tree

48. Define radius of a tree

49. Define center point of a tree

50. Define closure of G.

5 Marks

1. Prove that in any graph G the number of odd degree points of G is even.
2. Prove that any self complementary graphs has 4n or 4n+1 points.
3. Let G be a (p,q) graph all of whose points have degree k or k+1. If G has t>0 points of degree k, such that t = p(k+1)-2q
4. A (p,q) has t points of degree m and all other points of degree n such that (m-n)t + p n=2q.

5. Prove that $\delta \le 2q/p \le \Delta$.
6. Let G be a k regular bigraph with bipartition (V_1,V_2) and k>0. Prove that $|V_1| = |V_2|$
7. Prove that any self complementary graphs has 4n or 4n+1 points.
8. prove that $r(\bar{G})= r(G)$
9. Prove that $\alpha+ \beta = p$.
10. Prove that every graph is an intersection graph.
11. Let G be a (p, q) graph. Prove that L(G) is a (q_1,q_L) graph where $q_L =½\sum d_i^2-q$.
12. Prove that in a graph G, any u-v walk contain a u-v path.
13. Prove that if $\delta \ge k$, then G has a path of length k.
14. Prove that a closed walk of odd length contains cycle.
15. Prove that a graph G with p points and $\delta \ge p-1/2$ is connectd.
16. Prove that a graph G is connected iff for any partition of V into subsets V_1 and V_2 then is a line of G joining a point of V_1 to a point of V_2.
17. Prove that if G is not connected then $\bar{G}$ is connected.
18. Prove that a line x of a connected graph G is a bridge iff x is not any cycle of G.
19. Prove that every non-trivial connected graph n has at least two points which are not cut points.
20. Prove that for any graph G, $k \le \delta$.
21. If G is a graph in which the degree of every vertex is at least two then G contains a cycle.
22. Prove that every aamiltonian graph is 2-connected.
23. If G is Hamiltonian, then prove that for every non empty proper subsets of V(G), $\omega(G-S) \le |S|$ when ω denotes the no of components.
24. State and prove Dirac theorem.
25. Let G be a (p,q) graph with two non adjacent points u and v ∋ $d(u)+d(v)\ge p$. Then prove that G is Hamiltonian
26. Prove that c(G) is well defined.
27. Prove that graph is Hamiltonian iff its clouser is Hamiltonian.
28. Prove that Petersen graph is non Hamiltonian.
29. Prove that every connected graph has a spanning tree.
30. Prove that every tree has a centre consisting of either one point or two adjacent points.

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10 Marks.

1.The maximum number of lines among all p points graphs with no triangle is $[p^2/2]$.

2. Prove that $\alpha+ \beta = p = \alpha'+ \beta'$.

3. Let G_1 be a (p_1,q_1) graph and G_2 be a (p_2,q_2) graph then prove that

(i) $G_1 \cup G_2$ is a (p_1+p_2,q_1+q_2) graph.

(ii) G_1+G_2 is a $(p_1+p_2,q_1+q_2+p_1p_2)$ graph.

(iii) $G_1\times G_2$ is a $(p_1p_2,q_1p_2+q_2p_1)$ graph.

(iv) $G_1[G_2]$ is a $(p_1p_2,q_1p_2^2+q_2p_1)$ graph.

4. If A is the adjacency matrix of a graph with $V=\{v_1,v_2,v_3,\ldots\ldots v_p\}$,prove that for any $n \geq 1$ the $(i,j)^{th}$ entry of A^n is the number of v_i-v_j walks of length n in G.

5. A graph G with at least two points is bi partite iff all its cycles are of even length.

6. Prove that the following statements are equivalent.

(i).G is eulerian.

(ii). Every point of G has a even degree.

(iii). The set of edges of G can be partitioned into cycles.

7. Prove that the following statements are equivalent

(i). G is a tree.

(ii). Every two points of G are joined by a unique path.

(iii). G is connected and p = q+1.

(iv). G is acyclic and p = q+1.

8. LetG be a connected graph , . P.T the following statements are equivalent

(i) G is a block. (ii) any two points of G lie on a common cycle.

(iii) any point and a line of G lie on a common cycle. of G lie on a common cycle

(iv) any two lines of G lie on a common cycle.

9.State and prove Dirac theorem.

10. State and prove Chavatal theorem.

11. Let G be a connected graph , . P.T the following statements are equivalent

(i) u is a cut point of G.

(ii) there exists a partition of V-{u}into subsets Uand W such that for each x€ U and w€W the point v is on every cycle u-w path.

(iii) there exists two points u and w distinct from v such that v in on every u-w path.

12. A graph G with at least two points is a bipartite iff all its cycle are of even length.

13.Let G be a connected graph , . P.T the following statements are equivalent

(i) x is a bridge of G. (ii) there exists a partition of V-{u}into subsets U and W such that for each

u € U and w €W the line x is on every cycle u-w path.

(iii) there exists two points u and w distinct from v such that x is on every u-w path.

14. Define closure of a graph G and prove that c(G) well defined.
15. . Prove that Petersen graph is non Hamiltonian.
16. a). Let G be a (p,q) graph with two non adjacent points u and v ∋ $d(u)+d(v)\geq p$. Then prove that G is Hamiltonian

b) . Prove that graph is Hamiltonian iff its closure is Hamiltonian

17. Define centre of a tree G and Prove that every tree has a centre consisting of either one point or two adjacent points.

18. a) Prove that every graph is an intersection graph.

b) Define line graph and prove that $L(G)=\frac{1}{2}\sum di^2 - q$.

19. a) P.T every non trivial connected graph has atleast two points which are not cut points.

b) P.T the line x of a connected graph G is a bridge iff x is not an any cycle of G.

20. a) For any graph G, $\kappa \leq \lambda \leq \delta$.

b). If G is k-connected graph then $q \geq p.k/2$.

MATHEMATICS FOR COMPETITIVE EXAMINATIONS II- BSMA 57

UNIT :1 CHAIN RULE & TIME AND WORK

1.If 8 men can reap 80 hectares in 24 days ,Then how many hectares can 36 men reap in 30 days?

2.If 7 spiders makes 7 webs in 7 days. Then 1 spider will make 1 web in how many days?

3.Some persons can do a piece of work in 12 days. Two times the number of such persons will do half of that work?

4.If 15 men, working 9 hours a day , can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

5.If 15 toys cost Rs.234,What do 35 toys cost?

6.If the wages of 6 men for 15 days be Rs.2100,Then find the wages of 9 men for 12 days?

7.The price of 357 mangoes is Rs. 1517.25. What will be the approximate price of 9 dozens of such mangoes?

8.A can finish a work in 24 days ,B in 9 days and C in 12 days. B and C starts the work .But are forced to leave after 3 days. The remaining work was done by A ?

9.4men and 6 women can complete a work in 8 days, While 3 men and 7 women can complete it in 10 days .In how many days will 10 women complete it ?

10.A is twice as good as workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

11.A and B can do a work in 12 days, B and C in 15 days, C and A in 20 days .If A,B and C work together , they will complete the work?

12.A man can do a piece of work in 5 days ,but with the help of his son, he can do it in 3 days. In what time can the son do it alone?

13.A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work?

14.A sum of money is sufficient to pay A’s wages for 21 days and B’s wages for 28 days .The same money is sufficient to pay the wages of both?

15.10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?

16.Which of the following trains is the3 fastest?

17. A person crosses a 600 m long street in 5 minutes .What is his speed in km per hour?

18.A is twice as fast as B and B is thrice as fast as C is the journey covered by C in 54 minutes will be covered by B?

19.3 pumps, working 8 hours a day, can empty a tank in 2 days. How many hours a day must 4 pumps work to empty the tank in 1 day?

20. If the cost of x metres of wire is d rupees, then what is the cost of y metres of wire at the same rate?

21. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 minutes?

22. A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is

23. 39 persons can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day, complete the work?

24. A man completes $\frac{5}{8}$ of a job in 10 days. At this rate, how many more days will it takes him to finish the job?

25. If a quarter kg of potato costs 60 paise, how many paise will 200 gm cost?

26. In a dairy farm, 40 cows eat 40 bags of husk in 40 days. In how many days one cow will eat one bag of husk?

27. A wheel that has 6 cogs is meshed with a larger wheel of 14 cogs. When the smaller wheel has made 21 revolutions, then the number of revolutions mad by the larger wheel is:

28. If 7 spiders make 7 webs in 7 days, then 1 spider will make 1 web in how many days?

29. A flagstaff 17.5 m high casts a shadow of length 40.25 m. The height of the building, which casts a shadow of length 28.75 m under similar conditions will be:

30. In a camp, there is a meal for 120 men or 200 children. If 150 children have taken the meal, how many men will be catered to with remaining meal?

31. An industrial loom weaves 0.128 metres of cloth every second. Approximately, how many seconds will it take for the loom to weave 25 metres of cloth?

32. 4 mat-weavers can weave 4 mats in 4 days. At the same rate, how many mats would be woven by 8 mat-weavers in 8 days?

33. A can lay railway track between two given stations in 16 days and B can do the same job in 12 days. With help of C, they did the job in 4 days only. Then, C alone can do the job in:

34.Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio between the capacity of a man and a woman?

UNIT:2 TIME AND DISTANCE

1.A car is running at a speed of 108 kmph .What distance will it cover in 15 seconds?

2.Three persons are walking from a place A to another place B .Their speeds are in the ratio of 4:3:5.The time ratio to reach B by these persons will be?

3.A walks at 4 kmph and 4 hours after his starts, B cycles afte4r him at 10 kmph. How far from the start does B catch up with A?

4.In coveting a distance of 30 km, Abhay takes 2 hours more than sammeer.If Abhay doubles his speed , Then he would take 1 hour less than Sameer . Abhay’s speed?

5.Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph .For how many minutes does the bus stop per hour?

6. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minutes?

7. Two trains A and B start simultaneously in the opposite direction from two points P and Q and arrive at their destinations 16 and 9 hours respectively after their meeting each other. At what speed does the second train B travel if the first train travels at 120 km/h?

8. Two horses start trotting towards each other, one from A to B and another from B to A. They cross each other after one hour and the first horse reaches B, 5/6 hour before the second horse reaches A. If the distance between A and B is 50 km. what is the speed of the slower horse?

9. The speed of a car increases by 2 kms after every one hour. If the distance travelling in the first one hour was 35 kms. what was the total distance travelled in 12 hours?

10. A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. The average speed for the first 320 km of the tour ?

11. The distance from town A to town B is five miles. C is six miles from B. Which of the following could be the maximum distance from A to C?

12. A person goes to his office at 1/3rd of the speed at which he returns from his office. If the average speed during the whole trip is 12 m /h .what is the speed of the person while he was going to his office?

13. A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?

14. An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in $1\frac{2}{3}$ hours, it must travel at a speed of:

15. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is:

16. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is:

17. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

20. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is:

21. A man complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km/hr and second half at the rate of 24 km/hr. Find the total journey in km.
22. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 4 hours, then the speed of the first train is:
23. A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. The average speed for the first 320 km of the tour is:
24. A car travelling with $\frac{5}{7}$ of its actual speed covers 42 km in 1 hr 40 min 48 sec. Find the actual speed of the car.
25.It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the cars is:
26. Robert is travelling on his cycle and has calculated to reach point A at 2 P.M. if he travels at 10 kmph, he will reach there at 12 noon if he travels at 15 kmph. At what speed must he travel to reach A at 1 P.M.?
27. A farmer travelled a distance of 61 km in 9 hours. He travelled partly on foot @ 4 km/hr and partly on bicycle @ 9 km/hr. The distance travelled on foot is:

UNIT:3 PROBLEMS ON TRAIN

1.A train speeds past a pole in 15 seconds and a platform 100m long 25 seconds .Find its length?

2.A speed of 14 metres per second is the same as?

3.The length of the bridge ,Which a train 130 m long and travelling at 45 kmph can cross in 30 seconds ?

4.A train moves with a speed of 108 kmph. Its speed in metres per second?

5.A train 132 m long passes a telegraph pole in 6 seconds .Find the speed of the train ?

6. A man sitting in a train which is running at a speed of 100 km/hr saw a goods train which is running in opposite direction towards him. The goods train crosses the man in 8 seconds. If the length of goods train is 300 meters, find its speed.

7. Two trains of equal length are moving in same direction on parallel tracks at speed of 92 km/hr and 72 km/hr respectively. The faster train crosses the slower train in 18 seconds. Find the length of each train.

8. Two trains of length 140 meters and 166 meters are moving towards each other on parallel tracks at a speed of 50 km/hr and 60 km/hr respectively.In what time the trains will cross each other from the moment they meet?

9. Two trains running in opposite direction cross a man standing on the platform in 36 seconds and 26 seconds respectively. The trains cross each other in 30 seconds. What is the ratio of their speeds?

10. Two trains of length 120 meters and 140 meters are moving in the same direction on parallel tracks at speed of 82 km/hr and 64 km/hr. In what time the first train will cross the second train?

11. A train of length 200 meters takes 12 seconds to cross a man who is running at a speed of 10 km/hr in opposite direction of the train. What is the speed of the train?

12. Two trains are moving towards each other with speeds 40 km/hr and 45 km/hr from different stations P and Q. When they meet the second train from station Q has covered 20 km more distance than the first train which starts from station P. What is the distance between the two stations?

13. Two trains of length 125 meters and 115 meters are running on parallel tracks. When they run in the same direction the faster train crosses the slower train in 30 seconds and when they run in opposite direction they cross each other in 10 seconds. What is the speed of each train?

14. A train crosses two men who are running in the direction of train at 4 km/hr and 8 km/hr in 18 and 20 seconds respectively. Find the length of train.

15. A train moving at 108 km/hr crosses a platform in 30 seconds. Then it crosses a man running at 12 km/hr in the same direction of train in 9 seconds. What is the length of train and platform?

16. Two, trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is:

17. Two trains are running at 40 km/hr and 20 km/hr respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train?

18. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds respectively. The length of the train is:

19. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/hr. The other one walks at 5.4 km/hr. The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train?

20. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m. and travels towards B at 20 kmph. Another train starts from B at 8 a.m. and travels towards A at a speed of 25 kmph. At what time will they meet?

21. How many seconds will a 500 metre long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr?

22. Two trains are running in opposite directions with the same speed. If the length of each train is 120 metres and they cross each other in 12 seconds, then the speed of each train (in km/hr) is:

23. A train moves past a telegraph post and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train?

24. A 270 metres long train running at the speed of 120 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 seconds. What is the length of the other train?

25. Two trains of equal length are running on parallel lines in the same direction at 46 km/hr and 36 km/hr. The faster train passes the slower train in 36 seconds. The length of each train is:

26. A jogger running at 9 kmph alongside a railway track in 240 metres ahead of the engine of a 120 metres long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?

UNIT:4 BOATS AND STREAMS

1.A man can row upstream at 8 kmph and downstream at 13 kmph.

The speed of the stream?

2.A man can row upstream at 7kmph and downstream at 10 kmph. Find man rate in still water and the rate of current?

3.The speed of a boat in still water in 10 km/hr .If it can travel 26 km downstream and 14km upstream in the same time .What is the speed of the stream?

4.If a man rows at the rate of 5 kmph in still water and his rate against the current is 3.5 kmph. Then the mans rate along the current?

5.A man can row 18 kmph in still water .It takes him thrice as long to row up as to row down the river. Find the rate of stream?

6. A man swims 12 km downstream and 10 km upstream. If he takes 2 hours each time, what is the speed of the stream?

7. A boat covers 800 meters in 600 seconds against the stream and returns downstream in 5 minutes. What is the speed of the boat in still water?

8. A man can row a boat at a speed of 20 km/hr in still water. If the speed of the stream is 5 km/hr, in what time he can row a distance of 75 km downstream?

9. The speed of a boat in still water is 5km/hr. If the speed of the boat against the stream is 3 km/hr, what is the speed of the stream?

10. A man swimming in a river which is flowing at $3\frac{1}{2}$ km/hr finds that in a given time he can swim twice as far downstream as he can swim upstream. What will be his speed in still water?

11. A boat takes 6 hours to move downstream from point P to Q and to return to point P moving upstream. If the speed of the stream is 4 km/hr and speed of the boat in still water is 6 km/hr, what is the distance between point P and Q?

12. A motorboat travels 16 km in 2 hours against the flow of river and travels next 8 km along the flow of the river in 20 minutes. How long will it take motorboat to travel 48 km in still water?

13. A boat covers 6 km upstream and returns back to the starting point in 2 hours. If the flow of the stream is 4 km/hr, what is the speed of the boat in still water?

14. A man can row 9[1/3] km/hr in still water. He finds that it takes thrice as much time to row upstream as to row downstream (same distance). Find the speed of the current.

15. The velocity of a boat in still water is 9 km/hr, and the speed of the stream is 2.5 km/hr. How much time will the boat take to go 9.1 km against the stream?

16. A boatman goes 2 km against the current of the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will it take to go 5 km in stationary water?

17. A man can row three-quarters of a kilometre against the stream in $11\frac{1}{4}$ minutes and down the stream in $7\frac{1}{2}$ minutes. The speed (in km/hr) of the man in still water is:

18. Speed of a boat in standing water is 9 kmph and the speed of the stream is 1.5 kmph. A man rows to a place at a distance of 105 km and comes back to the starting point. The total time taken by him is:

19. A man rows to a place 48 km distant and come back in 14 hours. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. The rate of the stream is:

20. A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water?

21. The speed of a boat in still water in 15 km/hr and the rate of current is 3 km/hr. The distance travelled downstream in 12 minutes is:

22. A boat takes 90 minutes less to travel 36 miles downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 mph, the speed of the stream is:

23. A man can row at 5 kmph in still water. If the velocity of current is 1 kmph and it takes him 1 hour to row to a place and come back, how far is the place?

24. A boat covers a certain distance downstream in 1 hour, while it comes back in $1\frac{1}{2}$ hours. If the speed of the stream be 3 kmph, what is the speed of the boat in still water?

25. A motorboat, whose speed in 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is:

26. In one hour, a boat goes 11 km/hr along the stream and 5 km/hr against the stream. The speed of the boat in still water (in km/hr) is:

27. A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?

29. A boat can travel with a speed of 13 km/hr in still water. If the speed of the stream is 4 km/hr, find the time taken by the boat to go 68 km downstream.

UNIT :5 ALLIGATION AND MIXTURE

1.In what ratio must be mixed with milk to gain 20% by selling .Find mixture at cost price?

2.In what ratio must rice at Rs.9.30 per kg be mixed with rice at Rs.10.80 per kg .What is the mixture be worth Rs.10 per kg?

3.How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs.20.Find the mixture worth Rs.$10\frac{2}{3}$litre?

4.A dishonest milkman professes to sell his milk at cost price .But he mixes it with water and there by gains 25% . Find the percentage of water in the mixture?

5.In what ratio must water be mixed with milk costing Rs.12 per litre. To obtain a mixture worth of Rs.8 per litre?

6. A 60 liter mixture of milk and water contains 10% water. How much water must be added to make water 20% in the mixture?

7.700 ml of a mixture contains water and milk in the ratio 2:8. How much water must be added to the mixture so that the ratio of water and milk becomes 3:8?

8. A rice dealer bought 60 kg of rice worth Rs. 30 per kg and 40 kg of rice worth Rs. 35 per kg. He mixes the two and sells the mixture at Rs. 40 per kg. What is the percentage profit in this deal?

9. 1/2 and 1/4 parts of two bottles are filled with milk. The bottles are then filled completely with water and the content of bottles is poured into a container. Find the ratio of the milk and water in the container?

10. A bottle of whisky contains 40% alcohol. If we replace a part of this whisky by another whisky containing 20% alcohol, the percentage of alcohol becomes 28%. What quantity of whisky is replaced?

11. An alloy has copper and zinc in the ratio of 6:3 and another alloy has copper and tin in the ratio of 8:6. The equal weights of both the alloys are melted to form a new alloy. What will be the weight of tin per kg of the new alloy?

12. A shopkeeper mixes 60 kg of sugar worth Rs. 30 per kg with 90 kg of sugar worth Rs. 40 per kg. At what rate he must sell the mixture to gain 20%?

13. A 20 liter mixture contains 30% alcohol and 70% water. If 5 liters of water is added to the mixture, what will be the percentage of alcohol in the new mixture ?

14.Tea worth of Rs. 135/kg & Rs. 126/kg are mixed with a third variety in the ratio 1: 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be____?

15. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine the cask hold originally?

16. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

17. A can contains a mixture of two liquids A and B is the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

18. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?

19. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 kg?

20. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:

21. How many kilogram of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?

22. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

23. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is:

24. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg.

25. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10%?

26. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is:

27. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of water is 16 : 65. How much wine did the cask hold originally?

28. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is: